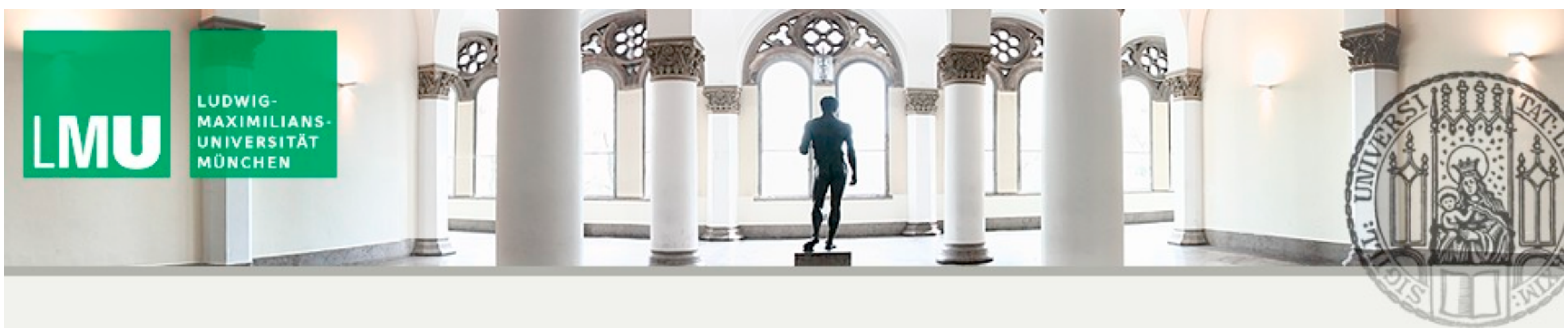
Black Holes and the Swampland

Dieter Lüst, LMU (ASC) and MPI München



Seminar Italy, 17. February 2022

Black Holes and the Swampland

Dieter Lüst, LMU (ASC) and MPI München

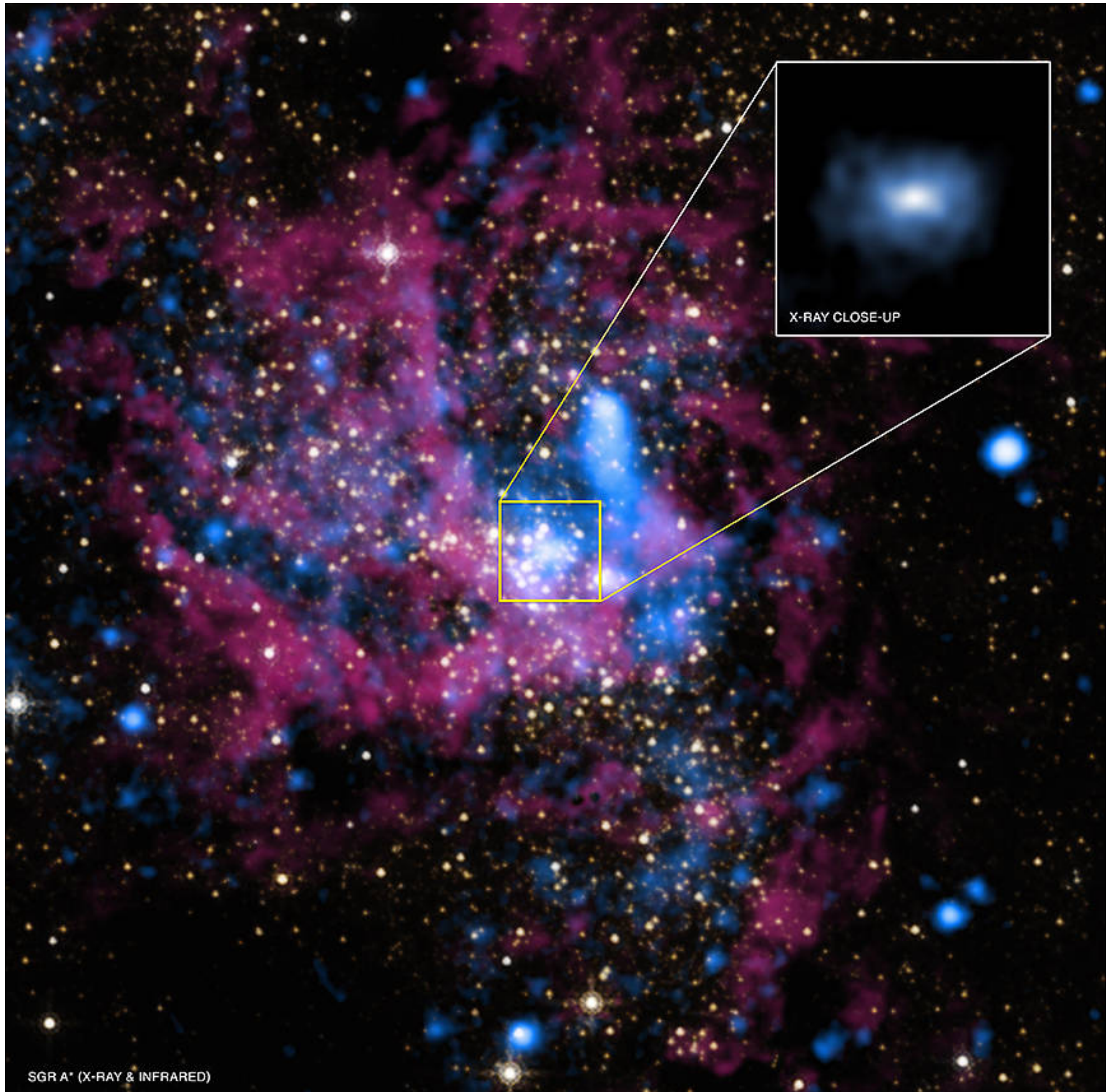
Work in collaboration with

Niccolo Cribiori, Markus Dierigl, Alessandra Gnecchi, Marco Scalisi,
arXiv:2202.04657

and also Quentin Bonnefoy, Luca Ciambelli, Severin Lüst,
Nucl. Phys. B958 (2020) 115112, arXiv:1912.07453

Seminar Italy, 17. February 2022

Black holes exist in nature !



Black Hole Sagittarius A*

Nobel prize in physics 2020:

Andrea Geht

Reinhard Genzel

Roger Penrose

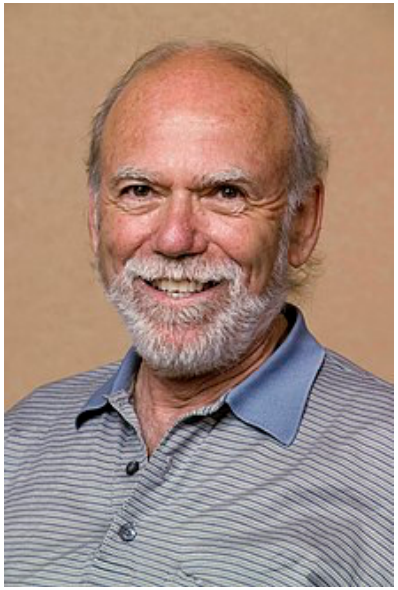
LIGO:

- Merging of binary black holes
- Discovery of gravitational waves

(Image credit: LIGO/A)

Nobel prize in physics 2017:

Rainer Weiss



Barry Barish

Kip Thorne

Event Horizon Telescope:

- Image of a Black Hole on April 10, 2019

Theory perspective:

No complete description of BHs, consistent with Quantum Mechanics !!

Two kinds of problems in Quantum Gravity:

- Singularities and infinities at $L_{\rm Planck}$

  Renormalizability of Quantum Gravity ?

- Puzzling physics around black hole horizon

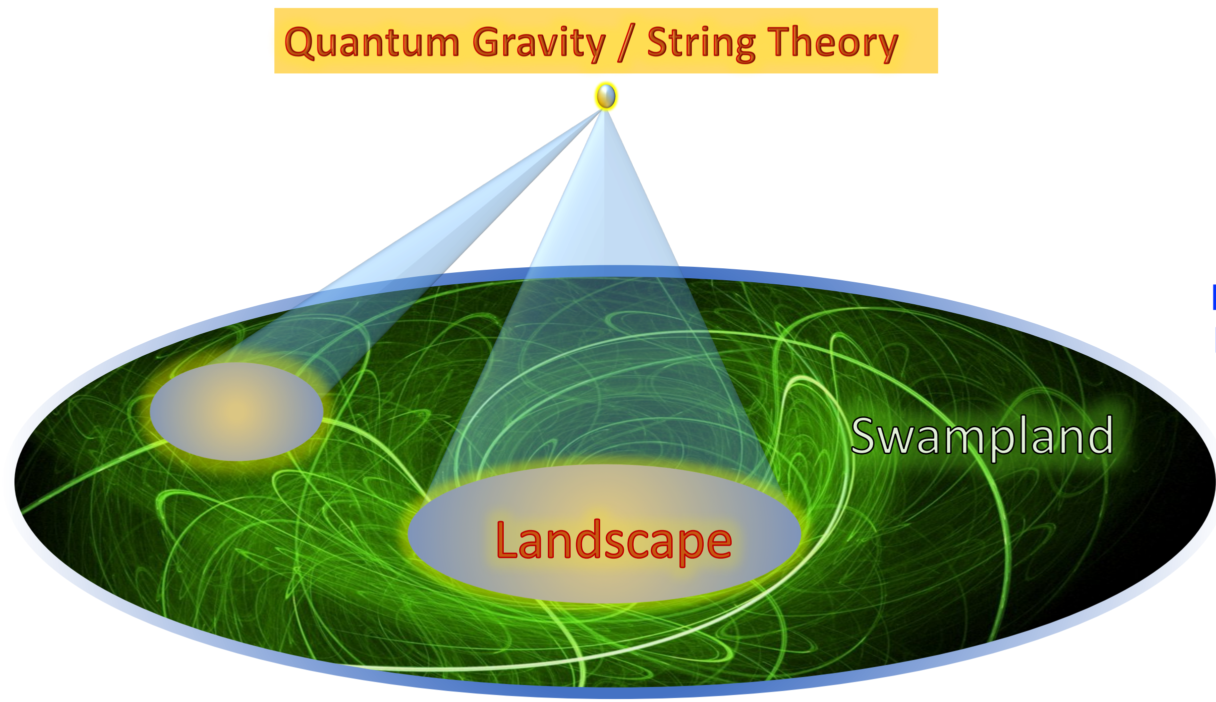
  Unitarity of Hawking radiation ?

Signal of breakdown of effective field theory ?

Need for UV completion of gravity ?

Swampland program:

Which IR consistent effective field theories (EFT) cannot be embedded into a UV complete quantum gravity theory?



[Reviews by E. Palti (2019)
M. van Beest, J. Calderon-Infante, D. Mirfendereski, I. Valenzuela (2021)]

[H. Ooguri, C. Vafa (2006)]

BHs play an important role in the swampland program !

Swampland distance conjecture (SDC):

At large distance Δ directions in the parameter space of string vacua there must be an infinite tower of states with mass scale m.

SDC: $$m = M_P e^{-\Delta}$$

[H. Ooguri, C.Vafa (2006)]

EFT breaks down at $\Lambda_{sw} \equiv m$

$\Lambda_{sw} << M_P$ when $\Delta \to \infty$

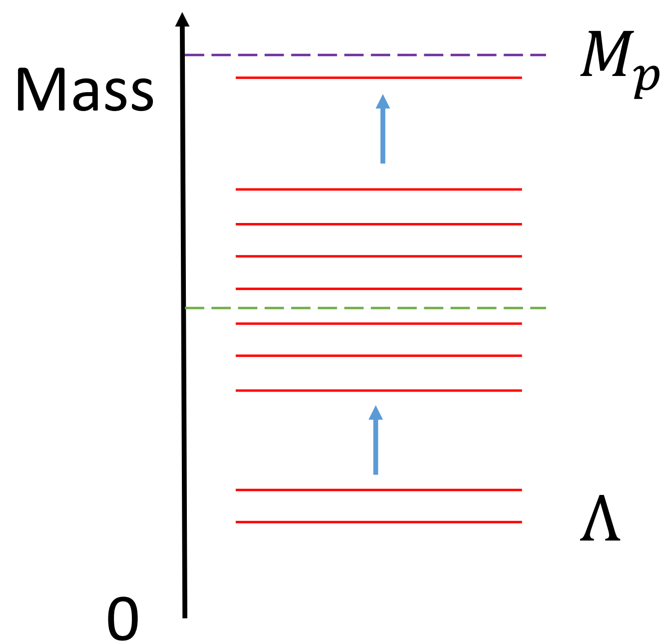
Decoupling condition: $mL_p > 0$ for $L_p \to 0$

Swampland arguments mostly apply when coupling gravity to additional matter fields.

For (string) compactifications the SDC is often due to the higher dimensional nature of theory:

At the KK mass scale a new dimension is opening up.

The relevant tower are the KK particles with masses



$$m_{KK} = \frac{n}{R}$$

or the winding strings with masses

$$m_{wind} = n' R$$

Related distance in the internal moduli space:

$$\Delta_R \simeq |\log R| \to \infty \quad \text{for} \quad R \to \infty, 0$$

In this talk we want to address the question if certain limits in the parameter spaces of black holes belong to the swampland and signal a break down of the effective field theory.

Basic two parameters:

Black hole entropy: $\mathcal{S}$

Black hole temperature: $\mathcal{T}$

Related distances in the BH moduli space: $\Delta_{\mathcal{S}}, \ \Delta_{\mathcal{T}}$

Interesting limits: large and small BHs with large/small entropy or temperature (possibly relevant for Hawking radiation).

For some additional discussion on properties and problems with small black holes see:

Y. Hamada, M.Montero, C. Vafa, I. Valenzuela (2021);
C. Holzhey, F. Wilczek (1992).

So we want to know what is happening in the limits where

$$\mathcal{T}, \mathcal{S} \to 0, \infty$$

and we ask

- $\Delta_{\mathcal{S}}, \Delta_{\mathcal{T}} \xrightarrow{?} \infty$

- Is there a related tower of light states (on BH horizon) in these limits ?

- Dualities between these limits?

Temperature and distance

Flat space-time with Euclidean time on a circle:

Temperature is given in terms of the radius β of time circle:

$$\mathcal{T} = \frac{\hbar}{\beta} = \frac{\hbar}{2\pi R_\tau}$$

Tower of states: Thermal KK modes = Matsubara modes

$$\omega_n = \frac{2\pi n}{\beta} = \frac{2\pi n}{\hbar}\mathcal{T}$$

Possible thermal winding modes: $\tilde{\omega}_{n'} \simeq \frac{n'}{\mathcal{T}}$ J. Atick, E. Witten(1988).

Temperature distance: $\Delta_{\mathcal{T}} \sim \left| \log\left(\frac{2\pi L_p}{\hbar}\mathcal{T}\right) \right|$

See also K. Saraikin, C. Vafa (2007)

Rindler space with acceleration parameter a:

$$ds^2 = (-a^2\rho^2 d\eta^2 + d\rho^2) + (dx_2)^2 + (dx_3)^2$$

Unruh temperature: $$\mathcal{T}_U = \frac{\hbar}{\beta_U} = \frac{\hbar a}{2\pi}$$

Unruh frequency: $$\omega_U = \frac{2\pi}{\hbar}\mathcal{T}_U = a$$

However for ω_U the swampland decoupling condition is no satisfied and the limit $\mathcal{T}_U \to 0$ should not lead to an inconsistent EFT.

The Schwarzschild black hole:

One parameter (= mass M) family of metrics:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\,, \quad f(r) = 1 - \frac{2MG_N}{r}$$

Horizon: $r_S = 2MG_N$ Surface gravity: $\kappa = (2r_S)^{-1}$

Bekenstein-Hawking entropy:

$$\mathcal{S} = \frac{8\pi^2 r_S^2}{L_p^2} = \frac{L_p^2 M^2}{2\hbar^2}$$

Hawking temperature:

$$\mathcal{T} = \frac{\hbar\kappa}{2\pi} = \frac{\hbar}{8\pi G_N M} = \frac{\hbar^2}{L_p^2 M}$$

For the Schwarzschild BH $\mathcal{S}$ and $\mathcal{T}$ are interdependent:

Hyperbola $\mathcal{S}\mathcal{T}^2 = 1/2$

Hawking modes: $\omega_H = \frac{2\pi}{\hbar}\mathcal{T} = \frac{\sqrt{2}\pi}{L_p\sqrt{\mathcal{S}}} = \frac{2\pi\hbar}{L_p^2 M}$

Limit of large blackhole:

$$\mathcal{T} \to 0\,, \mathcal{S} \to \infty \quad \Longrightarrow \quad \omega_H \to 0$$

Corresponding black hole distance: $\omega_H \simeq e^{-\Delta_{BH}}$

$$\Delta_{BH} = \left|\log\left(\frac{2\pi L_p}{\hbar}\mathcal{T}\right)\right| = \left|\log\left(\frac{\sqrt{2}\pi}{\sqrt{\mathcal{S}}}\right)\right| \underset{\substack{\mathcal{T}\to 0\\ \mathcal{S}\to\infty}}{\longrightarrow} \infty$$

Now the swampland decoupling condition is satisfied.

Δ_{BH} agrees with the distance in the space of Schwarzschild metrics:

Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019).

$$\Delta_g(\alpha) \sim \left| \int_{\alpha_i}^{\alpha_f} \mathrm{d}\alpha \left(\frac{1}{\mathrm{Vol}} \int d^4x \sqrt{g}\, g^{\mu\nu} g^{\rho\sigma} \frac{\partial g_{\mu\rho}}{\partial\alpha} \frac{\partial g_{\nu\sigma}}{\partial\alpha} \right)^{\frac{1}{2}} \right| \quad \simeq \log M$$

Questions:

What is the role of the Hawking modes for the swampland?

Does the (2d) EFT on the horizon break down in the limit $\mathcal{T} \to 0$ resp. $\mathcal{S} \to \infty$?

Is there a new tower of states in the limit of and $\mathcal{T} \to \infty$, $\mathcal{S} \to 0$, i.e. for small black holes?

The Reissner-Nordström charged black hole:

Two parameter (M, Q) family of metrics:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2, \quad f(r) = 1 - \frac{2MG_N}{r} + \frac{Q^2G_N}{r^2}$$

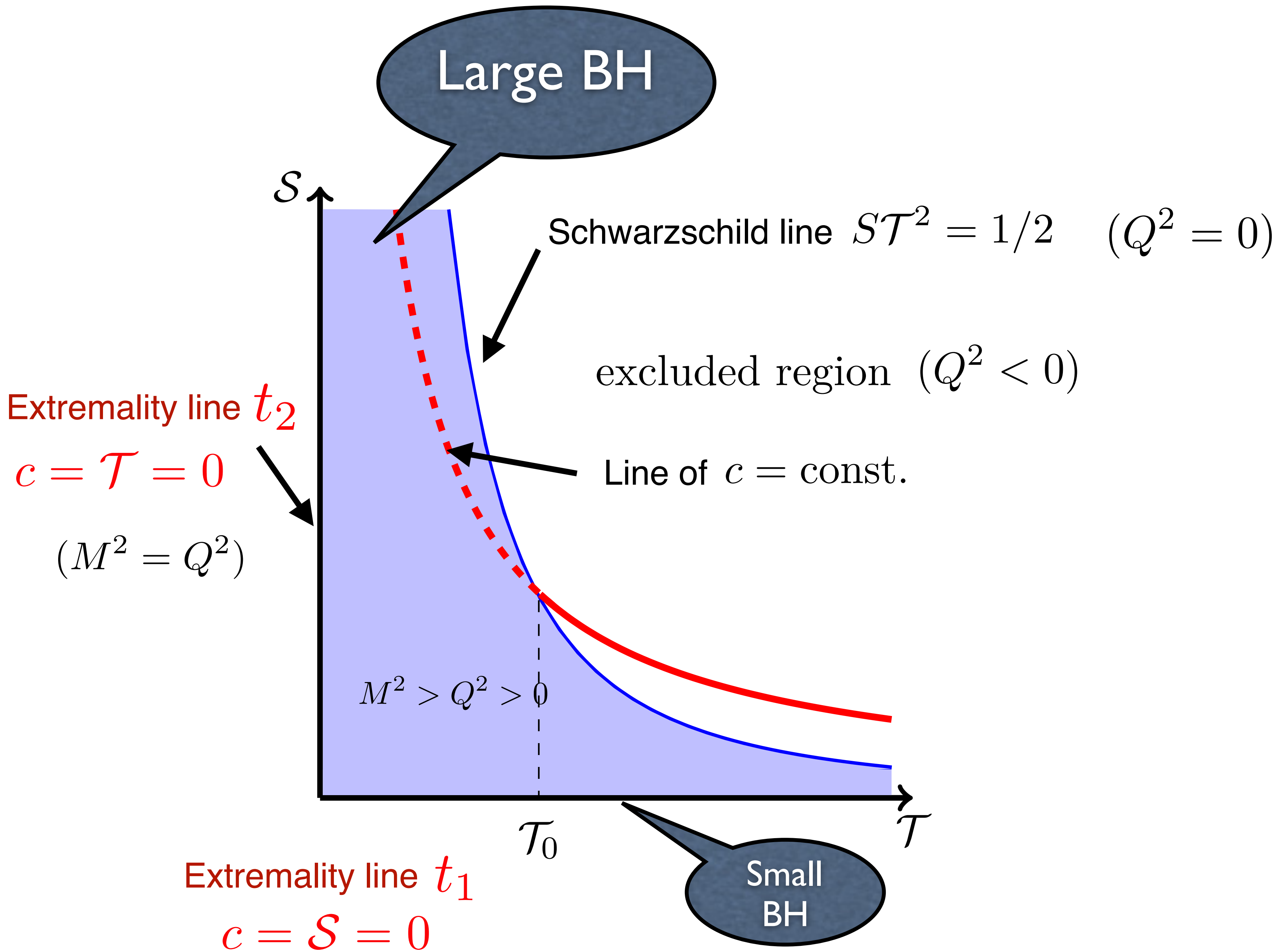
$$\mathcal{S} = \frac{8\pi^2 r_+^2}{L_p^2}$$

$$\mathcal{T} = \frac{\hbar\kappa}{2\pi} = \frac{\hbar c}{2\pi G_N (M+c)^2}$$

Extremality parameter: $c = \sqrt{M^2 - Q^2} = 2\mathcal{S}\mathcal{T}$

Hawking modes: $\omega_H = \dfrac{2\pi}{\hbar}\mathcal{T}$

Now $\mathcal{S}$ and $\mathcal{T}$ are independent parameters:



Metric distance in this parameter space:

$$\Delta(g) \simeq 2\sqrt{2}|\log \Delta \mathcal{T}| + \sqrt{2}|\log \Delta \mathcal{S}|$$

It becomes large for small/large $\mathcal{T}$ and $\mathcal{S}$.

In the extremal limit $\mathcal{T} \to 0$ a light tower of (Matsubara) modes appears.

(The limit $\mathcal{T} \to 0$ corresponds to the formation of an infinite throat with $AdS_2 \times S^2$ geometry.)

Does the SDC then predict a breakdown of the effective field theory for extremal BHs with $\mathcal{T} = 0$?

This seems to be quite unlikely !!

Therefore we want to couple the BHs to physical scalar fields: Dilaton ϕ , volume modulus $\mathcal{V}$.

Relate BH distance to moduli space distance:

$$\Delta_{\mathcal{S},\mathcal{T}} \iff \Delta_{\phi,\mathcal{V}}$$

Whenever $\Delta_{\phi,\mathcal{V}} \to \infty$ then the SDC predicts a breakdown of the 4d EFT.

The Electric/Magnetic dilaton black hole:

$$S_{\mathrm{EMd}} = \int d^4x\sqrt{-g}\Big(\tfrac{1}{2}M_p^2\big(R - \partial_\mu\phi\partial^\mu\phi\big) - \tfrac{1}{4}e^{-2\lambda\phi}F_{\mu\nu}F^{\mu\nu}\Big)$$

G. Gibbons, K. Maeda (1988); D. Garfinkle, G. Horowitz, A. Strominger (1991)

ϕ : Additional scalar fields, coupled to EM.

$$h = \frac{2}{1+2\lambda^2} \leq 2$$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\big(H_e H_m\big)^h dS_2^2\,,$$

$$f(r) = \big(H_e H_m\big)^{-h}\left(1 - \frac{c}{4\pi r}\right),$$

$$e^{-2\lambda\phi} = \left(\frac{H_e}{H_m}\right)^{2-h},$$

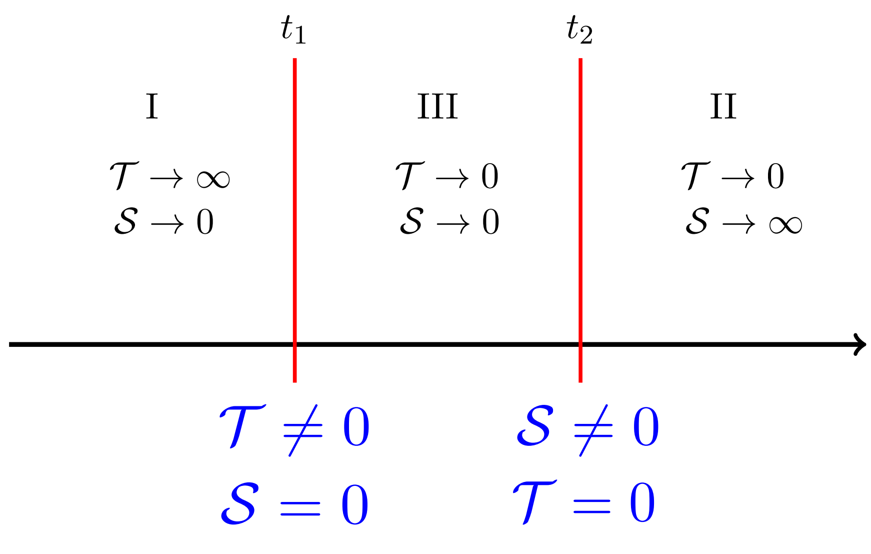
$$F = \frac{q}{r^2}H_e^{-2}H_m^{2-2h}dt\wedge dr + p\sin(\theta)d\theta\wedge d\varphi\,,$$

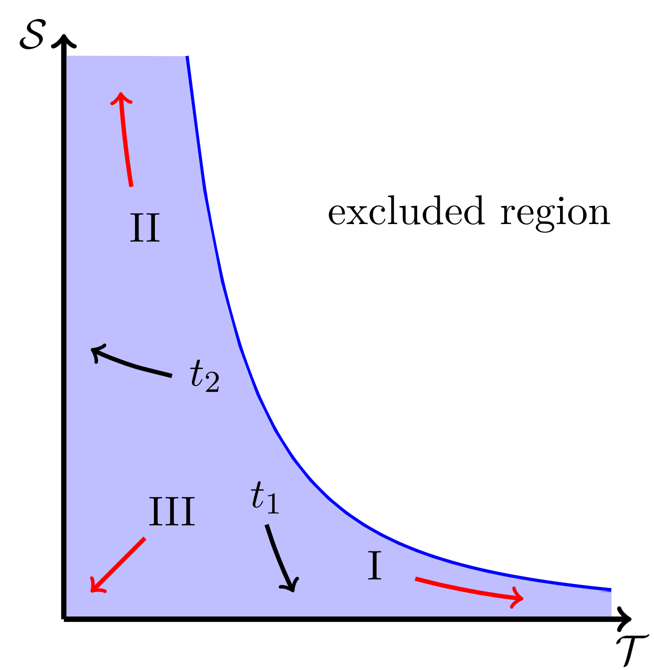
$$\mathcal{T} = \frac{1}{c}\left(H_e\left(\frac{c}{4\pi}\right)H_m\left(\frac{c}{4\pi}\right)\right)^{-h}$$

$$\mathcal{S} = \frac{1}{2}c^2\left(H_e\left(\frac{c}{4\pi}\right)H_m\left(\frac{c}{4\pi}\right)\right)^{h}$$

Extremal limit: $c = 2\mathcal{S}\mathcal{T} \longrightarrow 0$

Can be realised in different ways:





Single (magnetically) charged BH:

$$H_e(r) = 1\,, \quad H_m(r) = 1 + \frac{\xi_m}{r}\ , \ h\xi_m\big(\xi_m + \tfrac{c}{4\pi}\big) = p^2$$

2 kinds of extrema limits:

(i) magnetic RN with $h = 2$

$$t_2 : \mathcal{T} = 0, \mathcal{S} \neq 0 \ \Rightarrow \phi \rightarrow \text{const.}$$

$$\Delta_{\mathcal{T}} \rightarrow \infty \text{ but } \Delta_\phi \rightarrow \text{const.}$$

(ii) heterotic string with $h = 1$

M. Cvetic, D. Youm (1995)

$$t_1 : \mathcal{T} \neq 0, \mathcal{S} = 0 \ \Rightarrow \phi = \infty$$

$$\Delta_{\mathcal{S}} \simeq \Delta_\phi \simeq \log g(\phi) \sim \phi \rightarrow \infty$$

SDC $\Longrightarrow$ tower of light string states.

Dyonic BHs with electric charge q and magnetic charge p:

Extremal limit t_2 is at $\mathcal{T}=0$ and is at finite distance in scalar field space

$$\mathcal{S} \underset{c\to 0}{\longrightarrow} 8\pi^2 pq \qquad e^{-2\lambda\phi}\Big|_{r_+} \underset{c\to 0}{\longrightarrow} \frac{q}{p}$$

A $\quad q \text{ fixed}: \quad \mathcal{S} = \pi^2 q^2 e^{2\lambda\phi}\Big|_{r_+} \to \infty \quad \text{for} \quad \phi \to \infty$

B $\quad p \text{ fixed}: \quad \mathcal{S} = \pi^2 p^2 e^{-2\lambda\phi}\Big|_{r_+} \to \infty \quad \text{for} \quad \phi \to -\infty$

The infinite (zero) entropy limits are at infinite distance in the scalar field space and lead to a tower of massless states.

(Cases A and B are S-dual to each other.)

The N=2, multi-charged & multi-scalar black holes:

$\mathcal{N} = 2$ prepotential: $F(X^\Lambda) = \frac{1}{6} d_{ijk} \frac{X^i X^j X^k}{X^0}$

Type II A compactification on a torus or on a CY 3-fold:

Scalar fields: $z^i = X^i / X^0$ with $i = 1, \ldots, h_{1,1}$

U(1) gauge fields: F^Λ with $\Lambda = 0, \ldots, h_{1,1}$

Special case: STU - model $F(X) = \frac{X^1 X^2 X^3}{X^0}$

6-dimensional volume: $\mathcal{V} = -i z^1 z^2 z^3 = -iSTU$

KK states: $m_{KK} = \left(\frac{1}{\mathcal{V}}\right)^{1/6}$ Winding states: $m_{wind} = \mathcal{V}^{1/6}$

For a review see: L. Andrianopoli, R. D' Auria, S. Ferarrar, M. Trigiante (2008)

Non-extremal BHs with four non-vanishing charges:

$$q_0, p^1, p^2, p^3$$

$$ds^2 = -e^{2U}dt^2 + e^{-2U}\left[\frac{c^4 d\rho^2}{\sinh^4(c\rho)} + \frac{c^2}{\sinh^2(c\rho)}\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$e^{-2U} = e^{-2U_e + 2c\rho} \quad \text{with} \quad e^{-2U_e} = 4\sqrt{\mathcal{I}_4}$$

The 3 scalars run from spatial infinity to their values at the horizon $\rho = -\infty$.

Volume at horizon: $\mathcal{V}_h = \frac{a_0^2}{\sqrt{a_0 a^1 a^2 a^3}}$

$$\mathcal{I}_4 = I_0 I^1 I^2 I^3 \quad , \; I_0 = a_0 + b_0 e^{2c\rho}, \; I^i = a^i + b^i e^{2c\rho}$$

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \frac{1}{8\sqrt{2}L^0_\infty}\left[1 \pm \frac{1}{c}\sqrt{c^2 + 16 q_0^2 (L^0_\infty)^2}\right] \quad \begin{pmatrix} a^i \\ b^i \end{pmatrix} = -\frac{1}{8\sqrt{2}M_{i\,\infty}}\left[1 \pm \frac{1}{c}\sqrt{c^2 + 16 (p^i)^2 M_{i\,\infty}^2}\right]$$

Entropy: $$\mathcal{S} = 16\pi\sqrt{a_0 a^1 a^2 a^3}\, c^2$$

Temperature: $$\mathcal{T} = \frac{1}{32\pi}\frac{1}{c\sqrt{a_0 a^1 a^2 a^3}}$$

The known extremal (BPS) limit is obtained for $c = 0$

with $\mathcal{T} = 0\,, \quad \mathcal{S} = 2\pi\sqrt{q_0 p^1 p^2 p^3}$

$$\mathcal{V}_h = \frac{(q_0)^{3/2}}{\sqrt{p^1 p^2 p^3}}$$

K. Behrndt, G. Lopes Cardoso, B. De Wit,
R. Kallosh, D.L. , T. Mohaupt, (2008)

See A. Ceresole, S. Ferarra, A. Marrani (2010) for small N=2 extremal black holes.

See Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019) for the corresponding swampland discussion.

Consider now two non-extremal cases:

(A): q_0 kept fixed and p^i are varying.

$$\Longrightarrow \quad \boxed{\mathcal{V}_h = \frac{4\pi\mathcal{V}_\infty}{\mathcal{S}}\left(\mathcal{S}\mathcal{T} + \sqrt{\mathcal{S}^2\mathcal{T}^2 + \frac{2q_0^2}{\mathcal{V}_\infty}}\right)^2}$$

$$\mathcal{T} \to 0\,(\infty): \quad m_{KK} \simeq (q_0\mathcal{T})^{-1/3} \to \infty\,(0)$$

$$\mathcal{S} \to \infty(0) \qquad m_{wind} \simeq (q_0\mathcal{T})^{1/3} \to 0\,(\infty)$$

This leads to to the following BH temperature distance:

$$m = e^{-\Delta_\mathcal{T}} \quad \text{with} \quad \Delta_\mathcal{T} = \frac{1}{3}|\log(q_0\mathcal{T})| \quad \to \infty$$

(B): p^i kept fixed and q_0 is varying.

$$\Longrightarrow \quad \boxed{\mathcal{V}_h = \frac{8(M_\infty^1 M_\infty^2 M_\infty^3)^2 \mathcal{S}^3}{\pi^3 \prod_{i=1}^3 \left(\sqrt{4(M_\infty^i)^2(p^i)^2 + \mathcal{S}^2\mathcal{T}^2} + \mathcal{S}\mathcal{T}\right)^2}}$$

$$\mathcal{T} \to 0\,(\infty): \quad m_{KK} \simeq \mathcal{T}(p^1p^2p^3)^{1/3} \to 0(\infty)$$

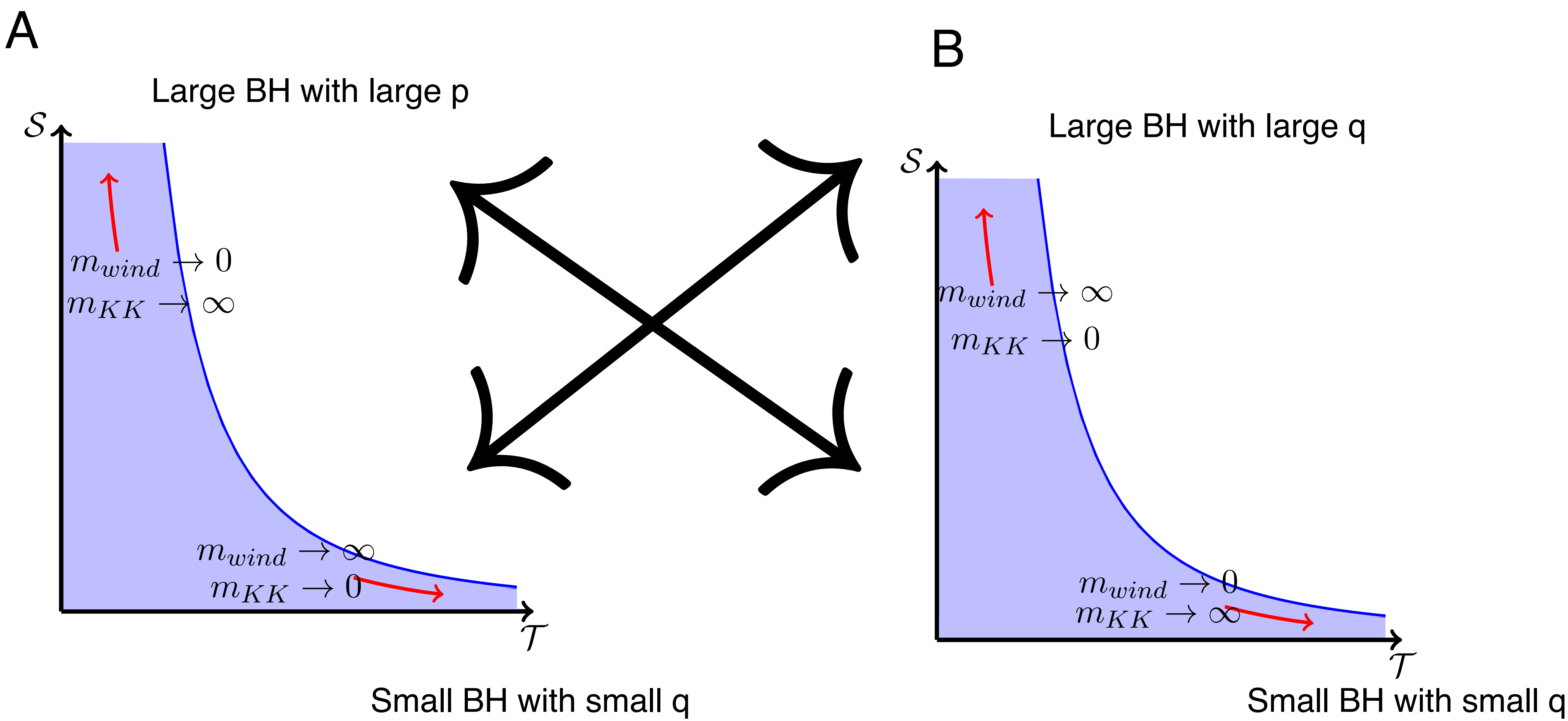
$$\mathcal{S} \to \infty(0) \quad m_{wind} \simeq \mathcal{T}^{-1}(p^1p^2p^3)^{-1/3} \to \infty(0)$$

This leads to to the following BH temperature distance:

$$m = e^{-\Delta\mathcal{T}} \quad \text{with} \quad \Delta_{\mathcal{T}} = \frac{1}{3}|\log(p^1p^2p^3\mathcal{T}^3)| \to \infty$$

Thermodynamic dualities between large and small BHs:

There is a dual behaviour between A and B type black holes: exchange of electric and magnetic charges.



Exchange of a large black hole (large p) with a small black hole (small q) leaves the internal KK and winding states unchanged!

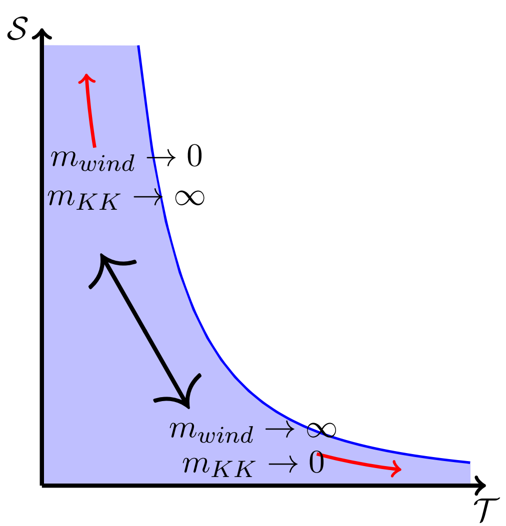
There are also dualities between large and small temperature/entropy inherited from T-duality in the internal compact space:

A

$$\mathcal{T} \longleftrightarrow \frac{1}{q_0^2 \mathcal{T}}$$

$$\mathcal{S} \longleftrightarrow \frac{q_0^4}{\mathcal{S}}$$

KK $\longleftrightarrow$ Winding

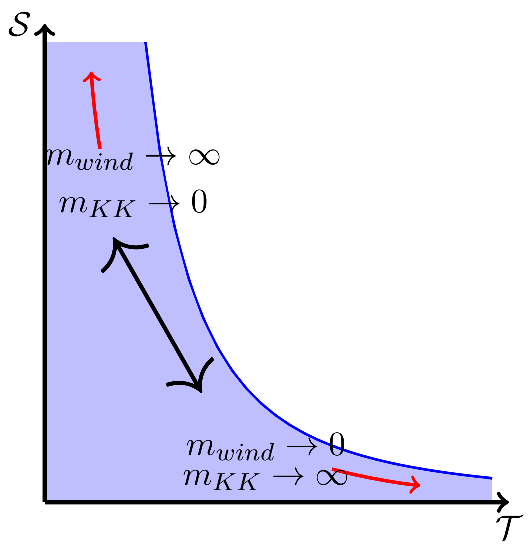


B

$$\mathcal{T} \longleftrightarrow \frac{1}{(p^1p^2p^3)^{2/3}\mathcal{T}}$$

$$\mathcal{S} \longleftrightarrow \frac{(p^1p^2p^3)^{4/3}}{\mathcal{S}}$$

KK $\longleftrightarrow$ Winding



The exchange of a small with a large black hole maps internal KK and winding states onto each other!

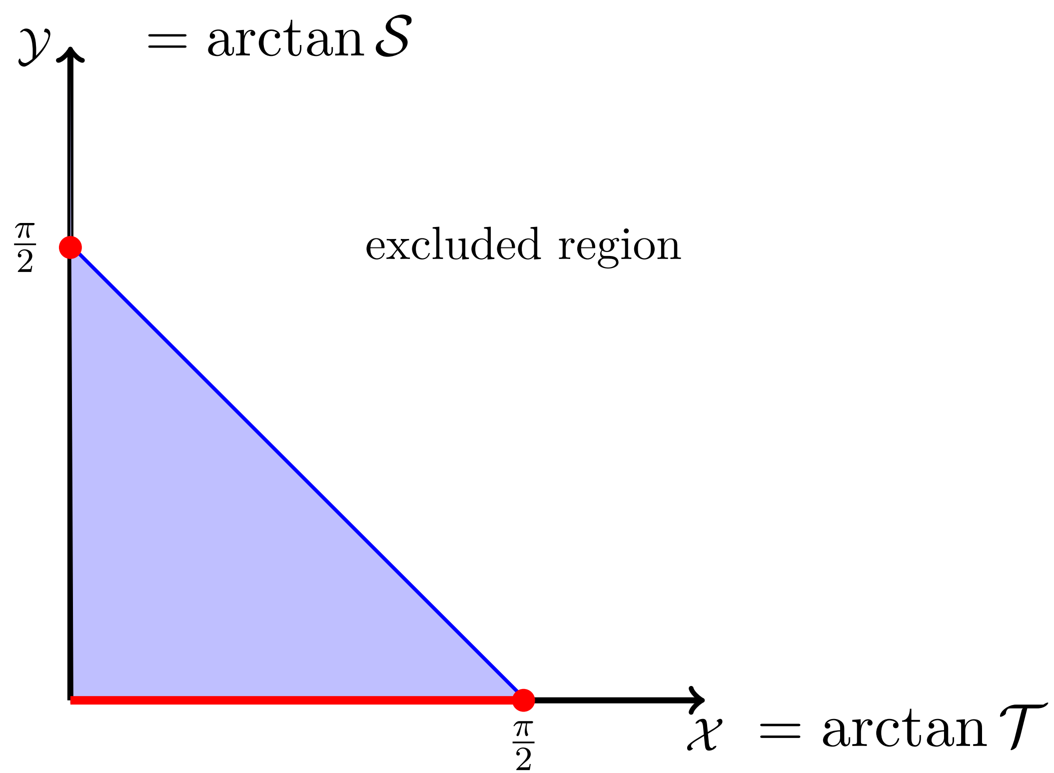
Summary

- Non-extremal, charged black holes with infinite/zero entropy resp. zero/infinite tenperature are at infinite distance in the black hole moduli space.
- Coupled to additional scalar fields these infinite distance limits can often lead to a tower of massless physical fields on the BH horizon and are therefore problematic from the SDC point of view.
  (This is not the case if electric and magnetic charges scale in the same way.)
- In particular small extremal black holes with vanishing entropy and constant temperature lead to a massless tower of states and are problematic.
  (Here quantum and higher curvature corrections seem to be important).

For some recent entropy arguments see: Y. Hamada, M.Montero, C. Vafa, I. Valenzuela (2021).

Summary

- However extremal black holes with vanishing temperature and finite entropy do not lead to a massless tower and are not doomed by the SDC.



- We have seen some interesting thermodynamic dualities between small and large black holes.

Many thanks!