



Probing Black Hole Microstates

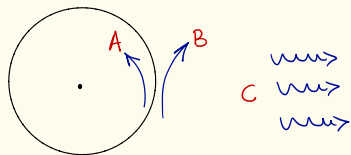
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October 18-19 2019

PRIN Kickoff Meeting - SNS

Overview

The Hawking paradox



Classical horizon \Rightarrow

- (i) AB are maximally entangled \Rightarrow
- (ii) A cannot be entangled with C \Rightarrow information loss !

Possible way outs

- Typical black hole microstates have a smooth horizon but there are **non-local effects** linking A to C \Rightarrow (ii) does not hold
(ER=EPR, Papadodimas-Raju, ...)
- **Effective field theory fails at distances of the order of the black hole horizon** and a typical microstate does not have a smooth horizon \Rightarrow (i) does not hold
(Fuzzballs, firewalls, ...)

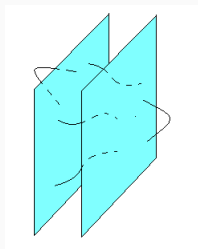
A holographic perspective

- In some situations, a black hole is dual to an ensemble in a 2D CFT

Black hole $\xrightarrow{\text{decoupling}}$ AdS₃ $\xleftarrow{\text{holography}}$ 2D CFT

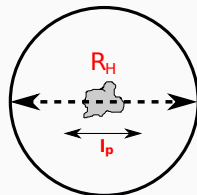
- A b.h. microstate is dual to a “heavy” operator O_H ($\Delta_H \sim c \gg 1$)
What is the description of O_H when $g_s^2 c \gg 1$?

$$g_s^2 c \ll 1$$



O_H

$$g_s^2 c \gg 1$$



ds_H^2

(EFT)

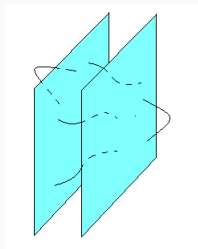
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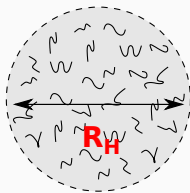
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(Fuzzball)

The fuzzball program

- Smooth geometries dual to **susy** b.h. microstates are known
- We have some (but limited) results for **non-susy** b.h.
- There are **non-trivial checks** of the duality between ds_H^2 and O_H
- The known geometries capture a **parametrically small** fraction of the entropy of b.h. with a classically macroscopic horizon

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Can typical b.h. microstates be described in supergravity?

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Can typical b.h. microstates be described in supergravity?

- Even if the answer is no, known microstates geometries encode non-trivial information on the CFT at strong coupling

Probing the microstates

- Microstates can be probed by “light” operators O_L ($\Delta_L \sim O(c^0)$)
- 4-point correlators

$$\langle \bar{O}_H(\infty) O_H(0) O_L(z) \bar{O}_L(1) \rangle \longleftrightarrow \langle O_L(z) \bar{O}_L(1) \rangle_{ds_H^2}$$

- They are non-protected and have informations on non-susy operators
- They diagnose information loss: they cannot decay at large t
- Correlators with O_H cannot be computed with Witten diagrams
- Witten diagrams in AdS_3 are subtle: no holographic correlator in a 2D CFT had been computed before
- In a certain limit: $\langle \bar{O}_H O_H O_L \bar{O}_L \rangle \rightarrow \langle \bar{O}_L O_L O_L \bar{O}_L \rangle$

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Microstate geometries provide an alternative method to compute holographic correlators

Plan of the talk

- The D1-D5-P black hole and the dual CFT
- Construction of the microstate geometries
- Holographic correlators and consistency with unitarity
- Outlook and open problems

The D-brane system

- The extremal 3-charge black hole in type IIB on $\mathbb{R}^{4,1} \times S^1 \times T^4$

$$D1_5 D5_{12345} P_5 \xrightarrow{\text{decoupling}} \text{AdS}_3 \times S^3 \times T^4 \longleftrightarrow \text{2D CFT}$$

with $\text{vol}(T^4) \sim \ell_s^4$ and $R(S^1) \gg \ell_s$

- The 2D CFT is the (4, 4) D1D5 CFT with $c = 6n_1 n_5 \equiv 6N \gg 1$
- The CFT has a 20-dim moduli space:
 - free orbifold point $\longleftrightarrow R_{\text{AdS}} \ll \ell_s$
 - strong coupling point $\longleftrightarrow R_{\text{AdS}} \gg \ell_s$

The D1-D5 CFT

- **Symmetries:**

(4,4) SUSY with $SU(2)_L \times SU(2)_R$ R-symmetry $\longleftrightarrow S^3$ rotations

The symmetry algebra is generated by:

$$L_n, J_n, G_{n+1/2}$$

- **The orbifold point:** sigma-model on $(T^4)^N/S_N$

The elementary fields are 4 bosons, 4 fermions and twist fields

- **Chiral primary operators:**

$$O_{(j,\bar{j})} \text{ with } h = j, \bar{h} = \bar{j}$$

(and their descendants with respect to the symmetry algebra) are protected: conformal dimensions and 3-point functions do not depend on the moduli

Microstate geometries

The graviton gas

- If O_k is a (anti)CPO of dimension k one can consider its descendants with respect to the global symmetry algebra

$$O_{k,m,n,q} \equiv (J_0^+)^m (L_{-1})^n (G_{-\frac{1}{2}}^{+1} G_{-\frac{1}{2}}^{+2})^q O_k$$

- “Semi-classical” states are coherent states

$$|B_1, B_2, \dots\rangle \equiv \sum_{p_1, p_2, \dots} (B_1 O_{k_1, m_1, n_1, q_1})^{p_1} (B_2 O_{k_2, m_2, n_2, q_2})^{p_2} \dots |0\rangle$$

- When $B_i^2 \sim N \gg 1$ the p_i -sum is peaked for $p_i \approx B_i^2/k$

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What is the gravitational description of $|B_1, B_2, \dots\rangle$?

Superstrata: construction

- $|0\rangle \longleftrightarrow \text{AdS}_3 \times S^3$
- Holography associates to O_k a sugra field $\phi_k : O_k \longleftrightarrow \phi_k$
- At linear order in B_i $|B_1, \dots\rangle$ is a perturbation of the vacuum

$$|0\rangle + B_i O_{k_i, m_i, n_i, q_i} |0\rangle \longleftrightarrow \text{AdS}_3 \times S^3 + B_i \phi_{k_i, m_i, n_i, q_i}$$

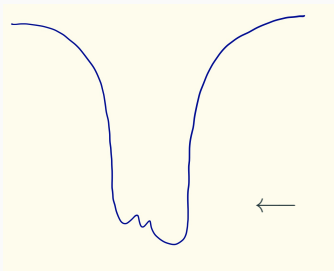
where $\phi_{k_i, m_i, n_i, q_i}$ solves the linearised sugra eqs. around $\text{AdS}_3 \times S^3$

$$\phi_{k, m, n, 0} = \frac{\rho^n}{(\rho^2 + 1)^{\frac{n+k}{2}}} \sin^{k-m} \theta \cos^m \theta e^{i[(k-m)\phi - m\psi + (k+n)\tau + n\sigma]}$$

- One can extend the linearised solution to an exact solution of the sugra eqs. valid for $B_i^2 \sim N$
- The non-linear extension is non-unique: ambiguities are fixed by imposing **regularity**

Superstrata: result

- The non-linear solutions are **smooth and horizonless**
- The solutions are asymptotically $\text{AdS}_3 \times S^3$ but in the interior AdS_3 and S^3 are non-trivially mixed
- The solutions can be glued back to flat space $\rightarrow \mathbb{R}^{4,1} \times S^1$ (after spectral flow to the R sector)
- There is a continuous family of solutions, parametrised by B_i , for fixed values of the global D1, D5, P charges



$$\mathbb{R}^{4,1} \times S^1$$

$$\text{AdS}_3 \times S^3$$

$$r \sim R_{\text{Hor}} \quad \text{no horizon!}$$

Holographic probes

- Consider

$$\langle O_L \rangle_H \equiv \langle \bar{O}_H(\infty) O_H(0) O_L(1) \rangle$$

with

- $O_H = \sum_{p_1, \dots} (B_1 O_{k_1, m_1, n_1, q_1})^{p_1} \dots \xleftrightarrow{\text{holography}} ds_H^2$

- $O_L = O_k \xleftrightarrow{\text{holography}} \phi_k$

- $\langle \bar{O}_H O_H O_L \rangle$ do not depend on the CFT moduli \Rightarrow
One can extract $\langle O_k \rangle_H$ from the geometry ds_H^2

$$\phi_k \xrightarrow{\rho \rightarrow \infty} \rho^{-k} \langle O_k \rangle_H$$

and compare with the value computed in the orbifold CFT

- What we learn:
 - Microstate geometries must have non-trivial multiple moments
 - Non-trivial checks of the sugra construction, including the non-linear completion

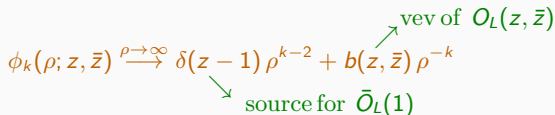
HLL correlators

- How to compute holographically

$$\mathcal{C}_H(z, \bar{z}) \equiv \langle \bar{O}_H(\infty) O_H(0) O_L(z, \bar{z}) \bar{O}_L(1) \rangle$$

- $O_L(z, \bar{z}) \equiv O_k(z, \bar{z}) \longleftrightarrow \phi_k(\rho; z, \bar{z})$
- Solve the linearised e.o.m. for ϕ_k in the background $ds_H^2 \longleftrightarrow O_H$
- Pick the non-normalisable solution such that
 - at the boundary ($\rho \rightarrow \infty$)

$$\phi_k(\rho; z, \bar{z}) \xrightarrow{\rho \rightarrow \infty} \delta(z-1) \rho^{k-2} + b(z, \bar{z}) \rho^{-k}$$



- in the interior ($\rho \rightarrow 0$) $\phi(\rho; z, \bar{z})$ is regular
- The correlator is given by

$$\mathcal{C}_H(z, \bar{z}) = \langle O_H | O_L(z, \bar{z}) \bar{O}_L(1) | O_H \rangle = b(z, \bar{z})$$

- We take

$$O_H = \sum_P (B O_1)^P \quad , \quad O_L = O_1$$

- O_H is a chiral primary $\Rightarrow P = 0$
- The ensemble of chiral primaries corresponds to a “small black hole” (massless limit of BTZ)

$$\frac{ds^2}{R_{\text{AdS}}^2} = \frac{d\rho^2}{\rho^2} + \rho^2(-d\tau^2 + d\sigma^2) + d\Omega_3^2$$

- The geometry ds_H^2 dual to O_H approximates the small black hole geometry in the limit $B^2 \rightarrow N$
- Computing \mathcal{C}_H for heavy states with $P \neq 0$ and finite B is harder, but see also [Bena, Heidmann, Monten, Warner](#)

Gravity

$$C_H = \alpha \sum_{l \in \mathbb{Z}} e^{il\sigma} \sum_{n=1}^{\infty} \frac{\exp \left[-i\alpha \sqrt{(|l| + 2n)^2 + \frac{(1-\alpha^2)l^2}{\alpha^2}} \tau \right]}{\sqrt{1 + \frac{1-\alpha^2}{\alpha^2} \frac{l^2}{(|l|+2n)^2}}$$

with $z = e^{i(\tau+\sigma)}$, $\bar{z} = e^{i(\tau-\sigma)}$, $\alpha = \left(1 - \frac{B^2}{N}\right)^{1/2}$

Free CFT

$$C_H = \frac{1}{|1-z|^2} + \frac{B^2}{2N} \frac{|z|^2 + |1-z|^2 - 1}{|1-z|^2}$$

The late time behaviour of the HLL correlator

- We focus on the limit $B^2 \rightarrow N \Leftrightarrow \alpha \rightarrow 0$
in which ds_H^2 approximates the “small b.h.”
- In this limit the series giving \mathcal{C}_H is dominated by terms with $n \gg \frac{|l|}{2\alpha}$:

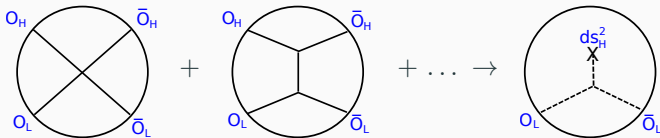
$$\mathcal{C}_H \sim \left[\frac{1}{1 - e^{i(\sigma - \tau)}} + \frac{1}{1 - e^{-i(\sigma + \tau)}} - 1 \right] \frac{\alpha}{1 - e^{-2i\alpha\tau}}$$

- The time-dependence of the correlator is controlled by α :
 - for $\tau \ll \alpha^{-1}$ one has $\mathcal{C}_H \sim \tau^{-1}$;
this is the same behaviour of the 2-point function in the “small b.h.”
 - for $\tau \gtrsim \alpha^{-1}$ \mathcal{C}_H stops decreasing with τ and oscillates
- Correlators in a pure or thermal state in a unitary theory with finite entropy do not vanish at late times

The late-time behaviour of \mathcal{C}_H is consistent with unitarity already at large c

A comment on the method

- Holographic correlators of single-trace operators (like O_L) are usually computed by summing Witten diagrams
- This technique has not been extended to correlators with multi-trace operators (like O_H)
- Even for single-trace correlators, the Witten-diagram method in AdS_3 has not been fully developed (the 4-point couplings are not known)
- Our approach bypasses Witten diagrams:



- For $B_k^2 \ll N$ heavy operators become light:

$$O_H \longrightarrow B_k O_k \equiv O_L \quad \text{for} \quad B_k^2 = 1$$

- Naively one expects

$$\langle \bar{O}_H(\infty) O_H(0) O_L(1) \bar{O}_L(z) \rangle \xrightarrow{B_k^2 \rightarrow 1} \langle \bar{O}_L(\infty) O_L(0) O_L(1) \bar{O}_L(z) \rangle$$

- This is not correct but it works for $z \rightarrow 1$:

the $B_k^2 \rightarrow 1$ limit of the HHLL correlator correctly captures all the single-trace operators exchanged between O_L and \bar{O}_L

- Using various consistency requirements (bootstrap) one can uniquely reconstruct $\langle \bar{O}_L O_L O_L \bar{O}_L \rangle$ from its $z \rightarrow 1$ limit

Summary and outlook

Results

- At strong coupling *some* heavy states in the black hole ensemble are described by smooth horizonless geometries
- HHL correlators can be used to construct and check the map between states and geometries
- Microstate geometries contain non-trivial informations on HHLL and LLLL correlators
- If probed for a short time microstates are indistinguishable from the black hole, but for sufficiently long times microstates deviate from the black hole and produce correlators that are consistent with unitarity already at large c
- These results are solid for *susy* states: there is a string-motivated mechanism to have non-trivial structure at the horizon scale

Open problems

- Classical supergravity works well for atypical states in the black hole ensemble
- For some observables, deviations from a typical state and the classical black hole should be exponentially suppressed in the entropy
- How much of our analysis can be extended to typical states?
- And what about microstates of non-BPS black holes?
- Can one make (semi)quantitative predictions that could be tested experimentally (GW, EHT)?
 - At which scale the geometry of a typical microstate starts to deviate from the classical black hole?
 - What is the dynamics controlling the interaction between a typical non-BPS fuzzball and infalling particles? How absorptive is the fuzzball surface?

Outlook

- Even if the general fuzzball paradigm is correct, it is possible that classical supergravity probes cannot resolve the structure of typical states
- Do we have quantitative tools to describe microstates beyond supergravity?
- Does one need to resort to full string theory?

(Massai, Martinec, Turton)

Or use insights from the CFT at strong coupling?

(Bootstrap, Lorentzian Inversion Formula, ...)