REVIEW TALK
(2+1)d dualities with $\mathcal{N} = 2$ supersymmetry

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Pisa, October 18, 2019

Outline

1. The contribution to the PRIN
2. Overview of the phases of (3+1)d SQCD
3. (2+1)d tools
4. (The) two main (2+1)d $\mathcal{N} = 2$ dualities
5. A new duality from compactifications: KK monopoles.
6. $\mathcal{I}_{S^3 \times S^1} \to \mathcal{Z}_{S^3}$
7. D-branes: reduction from T-duality.
8. Constructing the web

Motivations

Exchange of ideas and results between the **high-energy** and the **condensed-matter** communities, after the *discovery* of (2+1)d bosonization and it generalization to a more general **web**.

Project

Shed some light on the **web** of (2+1)d dualities drawing some inspiration from the knowledge of supersymmetric infrared dualities.

Techniques

This research will rely on: Supersymmetry breaking, Localization, Branes, . . .

Goals

- Tests and new 2+1 non-SUSY dualities;
- Generalization to other dimensions.

Today: review of (2+1)d dualities with four supercharges

Q Why $\mathcal{N} = 2$?

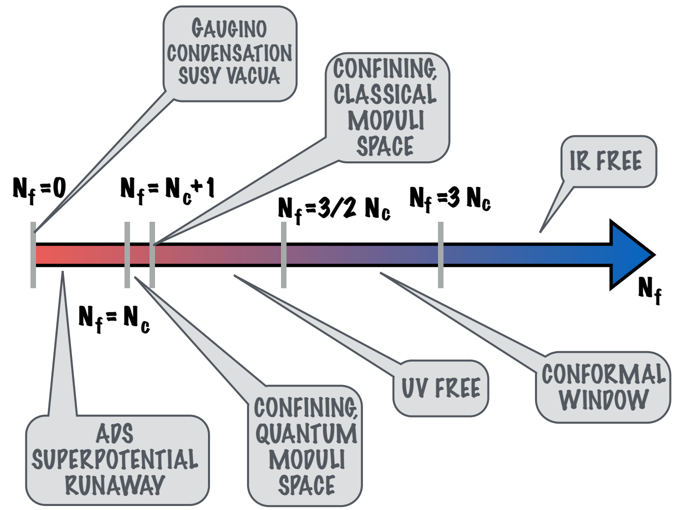
A1 Holomorphy protects W from quantum corrections; (for $\mathcal{N} = 1$ also time reversal needed).

A2 (2+1)d $\mathcal{N} = 2$ dualities can be derived from 4d $\mathcal{N} = 1$ by a "sensible" compactification.

A3 There is a large web of dualities (here we are focusing on for (2+1)d $U(N_c)$ SQCD).

This web will be the subject of the talk, we will show how to obtain it using (3+1)d results, dimensional reduction, brane engineering and localization (on the three sphere).

The phase structure of 4d SQCD



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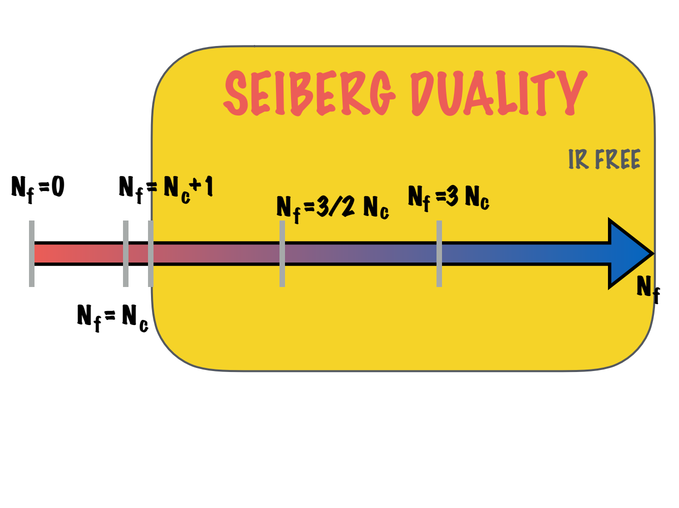


Tools in (2+1)d

SUSY offers many tools to study (2+1)d models and check conjectured dualities

- Moduli space: HB and CB
- CB coordinates, monopoles and "superpotentials"
- Localization: spheres, indices and topological twist
- Anomalies: gauge vs global, continuous vs discrete (parity)
- CS vs YM action
- Real masses and background symmetries
- Topological symmetry

Multiplets

Vector: $V = (A_\mu, \lambda_\alpha, \sigma, D)$ where σ from dim. red. of A_3
Chiral : $\Phi = (\phi, \psi, F)$

Coulomb branch (CB)

Due to $\langle\sigma\rangle$, combined with the dual photon $\varphi = d * F$
Chiral $\Sigma_i = \frac{\sigma_i}{g_3^2} + i\varphi_i$; e^{Σ_i} CB coordinate (UV monopole)
Monopole superpotentials: $W \propto e^{f(\Sigma_i)}$, lift some CB directions

Abelian global symmetries

Axial $U(1)_A$ (anomalous in (3+1)d); $U(1)_R$ R-symmetry;
topological $U(1)_J$ shifting φ.

Chern-Simons (CS) action

$S_{CS} = \frac{k}{4\pi} \int Tr(A \wedge dA - \frac{2}{3} A^3 - \lambda \tilde{\lambda} + 2\sigma D)$ w/ $k \in Z$

Real masses

Coupling $|\sigma^i_{bckg} T^i_R \phi_R|^2$, $\langle \sigma_{bckg} \rangle$ real mass for ϕ

CS and fermions

Integrating out fermions with large real masses generates an effective CS: $k^{eff}_{ij} = k_{ij} + \frac{1}{2} \sum_I c_i(\psi_I) c_j(\psi_I) sgn(m_I)$

Aharony duality '98

ELECTRIC: 3d $\mathcal{N} = 2$ $U(N_c)$ SQCD, with $N_f (> N_c)$ $\square$ Q and N_f $\overline{\square}$ $\tilde{Q}$;

$$W = 0$$

MAGNETIC: 3d $\mathcal{N} = 2$ $U(N_f - N_c)$ SQCD, with N_f $\square$ q and $\overline{\square}$ $\tilde{q}$, $M = Q\tilde{Q}$, $V_\pm$ monopoles of $U(N_c)$ (singlets of the dual phase) and $v_\pm$ monopoles of $U(N_f - N_c)$

$$W = Mq\tilde{q} + v_+ V_+ + v_- V_-$$

If $N_f = N_c = 1$ it reduces to mirror symmetry: $U(1)$ with $N_f = 1$ dual to 3 chirals interacting through $W = XYZ$

Giveon-Kutasov duality '08

ELECTRIC: 3d $\mathcal{N} = 2$ $U(N_c)_k$ SQCD, with $N_f + |k| > N_c$ and N_f $\square$ Q and and N_f $\overline{\square}$ $\tilde{Q}$;

$$W = 0$$

MAGNETIC: 3d $\mathcal{N} = 2$ $U(N_f - N_c + |k|)_{-k}$ SQCD, with N_f $\square$ q and $\overline{\square}$ $\tilde{q}$ and $M = Q\tilde{Q}$

$$W = Mq\tilde{q}$$

- 4d/3d reduction of $U(N_c)$ SQCD [Aharony, Razamat, Willett, Seiberg '13]
- On S^1 effective description with $W_{BPS-monopole}$ and $W_{KK-monopole}$ (reproduce SUSY vacua at $N_f = 0$)
- If $N_f \neq 0$: W_{KK}^{ele} and W_{KK}^{mag} participate to a new duality:

ARSW duality '13

ELECTRIC: 3d $\mathcal{N} = 2$ $U(N_c)$ SQCD, with $N_f (> N_c)$ $\square$ Q and N_f $\overline{\square}$ $\tilde{Q}$

$$W = W_{KK}^{ele} = \eta V_+ V_-$$

with $\eta = e^{-1/(r g_3^2)} = e^{-1/g_4^2} = \Lambda_{holo}^b$
MAGNETIC: 3d $\mathcal{N} = 2$ $U(N_f - N_c)$ SQCD, with N_f $\square$ q and N_f $\overline{\square}$ $\tilde{q}$

$$W = M q \tilde{q} + \tilde{\eta} v_+ v_-$$

- $U(1)_A$ broken (as in 4d) by KK monopoles
- GK and A from this new duality by real mass flows and Higgsing

This procedure can be reproduced by

Localization

Here we focus on the S^1 reduction of the 4d superconformal index:

$$\mathcal{I}^{ele}_{S^3 \times S^1_r} = \mathcal{I}^{mag}_{S^3 \times S^1_r} \underset{r \to 0}{\to} \mathcal{Z}^{ele}_{S^3} = \mathcal{Z}^{mag}_{S^3}$$

This procedure requires the cancellation of divergent pre-factors. It is not guaranteed to work (e.g. $\mathcal{N} = 4$ SYM and $SO(N_c)$ dualities).

Localization

SCI $= Tr(-1)^F e^{-\beta E} p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{i\in F} \mu_i^{q_i}$

$$I_{U(N_c)}^{(N_f,N_f)} = \frac{(p;p)^{N_c}(q;q)^{N_c}}{N_c!} \int \prod_{i=1}^{N_c} \frac{dz_i}{2\pi i z_i} \frac{\prod_{a=1}^{N_f} \Gamma_e((pq)^{\frac{R}{2}}\mu_a z_i)\Gamma_e((pq)^{\frac{R}{2}}\nu_a z_i^{-1})}{\prod_{i<j}\Gamma_e((z_i/z_j)^{\pm 1})}$$

with $(p;p) = \prod_{a=1}^{\infty}(1-p^{a+1})$ and $\Gamma_e(x) \equiv \prod_{k,m=0}^{\infty} \frac{1-p^{k+1}q^{k+1}/z}{1-p^k q^m z}$

Reduction: define
$p = e^{2\pi i r\omega_1}$, $q = e^{2\pi i r\omega_2}$, $\mu_a = e^{2\pi i r m_a}$, $\mu_a = e^{2\pi i r n_a}$, $z_i = e^{2\pi i r\sigma_i}$
The reduction corresponds to the limit $r \to 0$ with ω_i, m_a, n_a fixed.

$$\lim_{r\to 0} \Gamma_e((pq)^{\frac{r}{2}\mu_a z_i} \propto \Gamma_h(\frac{\omega_1+\omega_2}{2}R + m_a + \sigma_i)$$

Subtraction of a divergent term $\propto$ Tr R and Tr F

Localization

The 4d compact integral is now a non-compact integral over σ

$$Z_{G;k}(\lambda;\vec{\mu}) = \frac{1}{|W|}\int \prod_{i=1}^{G} \frac{d\sigma_i}{\sqrt{-\omega_1\omega_2}} e^{\frac{ik\pi\sigma_i^2}{\omega_1\omega_2}+\frac{2\pi i\lambda\sigma_i}{\omega_1\omega_2}} \frac{\prod_I \Gamma_h\left(\omega\Delta_I+\rho_I(\sigma)+\widetilde{\rho}_I(\mu)\right)}{\prod_{\alpha\in G_+}\Gamma_h\left(\pm\alpha(\sigma)\right)}$$

- Balancing condition: relations between the fugacities (3+1)d (due to anomalies) or (2+1)d real masses (due to W_{mon});
- FI: real mass for 3d $U(1)_J$ (here λ);
- Dualities as integral identities (subtleties due to ∞);
- Real mass flow and CS: $\lim_{x\to\infty}\Gamma_h(x) = e^{i\pi sgn(x)(x-\omega)^2}$;
- Flows to generate new dualities
- Holomorphic mass $\Gamma_h(x)\Gamma_h(2\omega-x) = 1$

Aharony

$$
\begin{aligned}
Z_{U(N);0}(\lambda;\mu;\nu) &= \prod_{a,b=1}^{F} \Gamma_h(\mu_a+\nu_a) Z_{U(F-N);0}(-\lambda;\omega-\nu;\omega-\mu) \\
&\times \Gamma_h\left(\frac{1}{2}(\sum_{a=1}^{F}(\mu_a+\nu_a) \pm \lambda) + \omega(F-N+1)\right)
\end{aligned}
$$

Giveon-Kutasov

$$
Z_{U(N);k}(\lambda;\mu;\nu) = e^{\phi} \prod_{a,b=1}^{F} \Gamma_h(\mu_a+\nu_a) Z_{U(F-N+|k|);-k}(-\lambda;\omega-\nu;\omega-\mu)
$$

where $\phi = \phi(\omega,\mu,\nu,\lambda,F,N,k)$ collects the (CS) contact terms.

ARSW

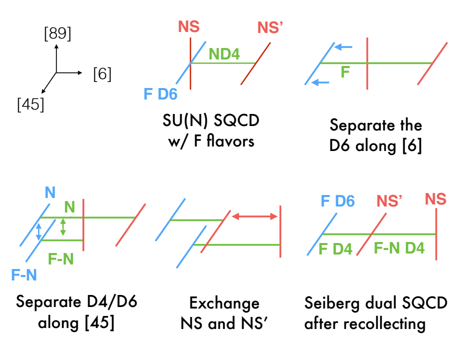
$$
Z_{U(N);0}(\lambda;\mu;\nu) = e^{\phi} \prod_{a,b=1}^{F} \Gamma_h(\mu_a+\nu_a) Z_{U(F-N);0}(-\lambda;\omega-\nu;\omega-\mu)
$$

with $\sum_{a=1}^{F}(\mu_a+\nu_a) = 2\omega(F-N)$ (balancing condition, enforced by W_{KK}).

D-branes

Duality due to the brane creation effect during the *transition through infinite coupling* (i.e. crossing of NS branes)

4d Seiberg duality and branes



D-branes: reduction from T-duality

	0	1	2	3	4	5	6	7	8	9
NS	X	X	X	X	X	X				
NS'	X	X	X	X					X	X
N_c $D4$	X	X	X	X			$[X]$			
N_f $D6$	X	X	X	X				X	X	X

T-duality $\rightarrow$ along x_3

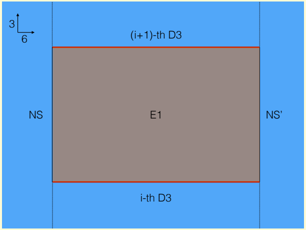
	0	1	2	3	4	5	6	7	8	9
NS	X	X	X	X	X	X				
NS'	X	X	X	X					X	X
N_c $D3$	X	X	X				$[X]$			
N_f $D5$	X	X	X					X	X	X

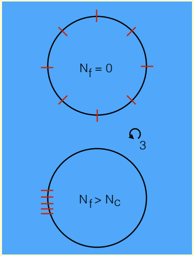
For $N_f = 0$ (SYM):

$W = W_{mon} = W_{mon}^{(BPS)} + W_{mon}^{(KK)}$

E1: Euclidean D1,

For $N_f \neq 0$ ($> N_c$) $W = W_{mon}^{(KK)}$





$$W \propto e^{\frac{\sigma_i - \sigma_{i+1}}{g_3^2} + i(\varphi_i - \varphi_{i+1})} + \eta e^{\frac{\sigma_N - \sigma_1}{g_3^2} + i(\varphi_N - \varphi_1)}$$

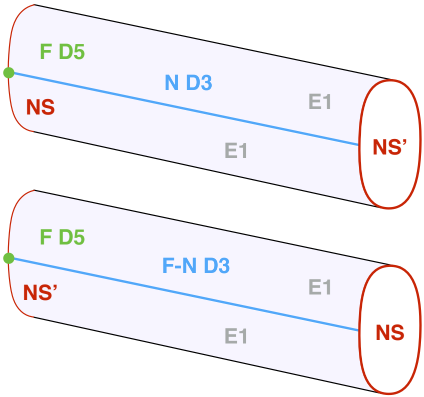
with Nambu-Goto (for σ) and Boundary action (for φ)

Results

[A.A., D.Forcella,C.Klare,D.Orlando,S.Reffert '15]
[A.A.,C.Klare,D.Orlando,S.Reffert '15]

- ARSW from T-duality + Hanany-Witten transition
  - KK monopole (from the affine root)
  - Orthogonal and symplectic gauge groups,
  - Tensorial matter with power low superpotential
- Brane picture of Aharony duality (monopole superpotential from *local* mirror symmetry)
- Giveon-Kutasov duality from Aharony duality on the brane picture

ARSW duality



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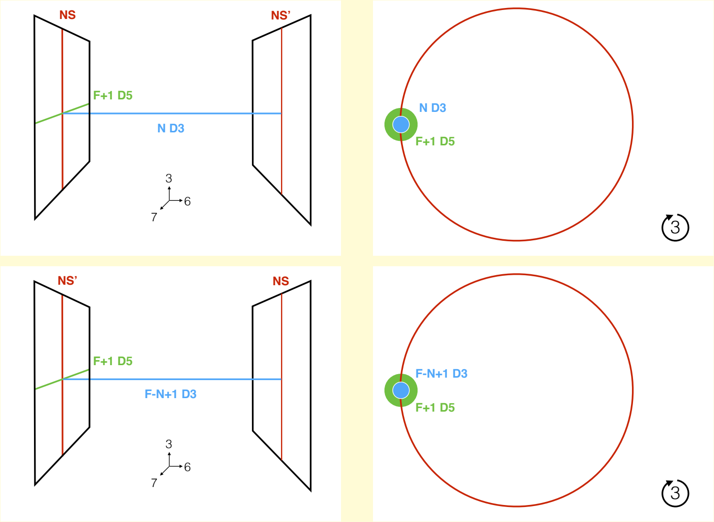
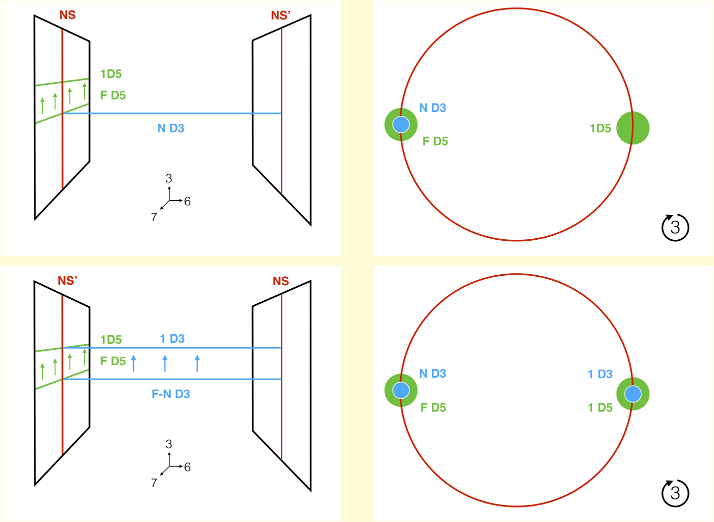
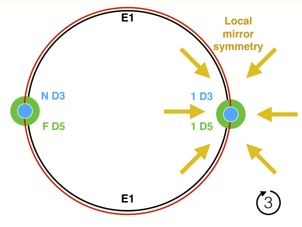
Aharony duality (I): ARSW duality with one extra flavor



Aharony duality (II): real mass flow and dual Higgsing



Aharony duality (III): local mirror symmetry



$W = Mq\tilde{q} + \hat{M}\hat{q}\hat{\tilde{q}} + e^{\Sigma_1 - \hat{\Sigma}} + e^{\hat{\Sigma} - \Sigma_N}$

Last two terms from $E1$

Local mirror symmetry at $x_3 = \pi r$

$W = Mq\tilde{q} + \hat{M}X + XYZ + e^{\Sigma_1}Y + Ze^{-\Sigma_N}$

Integrating out the massive fields

$W = Mq\tilde{q} + v_+ V_+ + v_- V_-$

where we used the standard identification

$$v_+ = X, \quad v_- = Y, \quad V_+ = e^{\Sigma_1}, \quad V_- = e^{-\Sigma_N}$$

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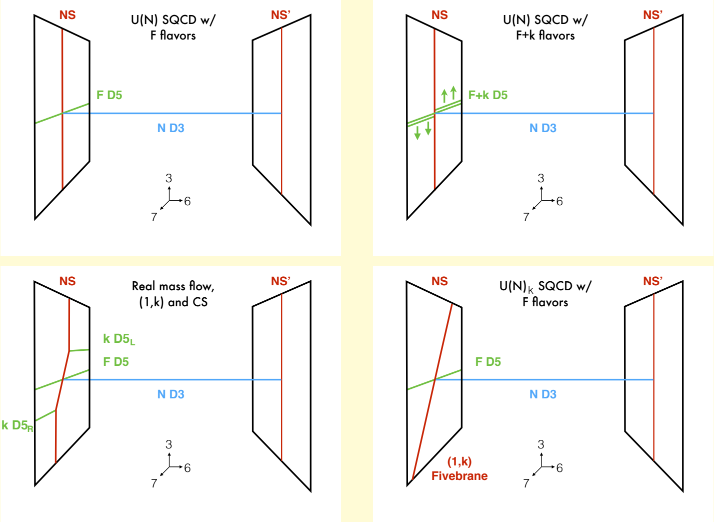
Real mass flow and CS from the branes

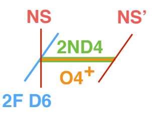


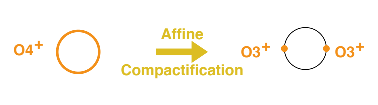
Other mechanisms

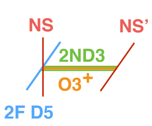
- Real mass flow generating "chiral" dualities, i.e. $F_L \neq F_R$, [Benini,Cremonesi,Closset '11]
- Higgs flow from the circle reduction of the Intriligator-Pouliot duality for $USp(2N_c)$ to $U(N_c)$: linear monopoles superpotentials, [Benini,Benvenuti,Pasquetti '17]
- Higher monopole powers and classification using systems of D-branes and O-planes (affine and twisted affine compactifications) [A.A,Garozzo, Mekareeya,'18] [A.A,Cassia,Garozzo, Mekareeya,'19]

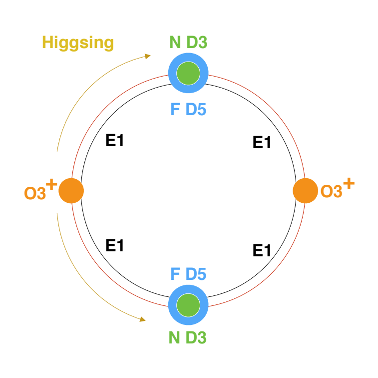
Note:
Each case has been checked by matching $\mathcal{Z}_{S^3}$ and the brane picture, describing the duality, has been found

BBP and branes









Consider 4d IP electric phase on S^1

$O4^+$ splits into the pair $(O3^+, O3^+)$ (Affine case)

Higgs flow: $USp(2N) \to U(N)$

$W_{mon} = V_+ + V_-$ generated by the E1

In general:

	$O4^+$	$O4^-$	$\widetilde{O4}^-$
A	$(O3^+, O3^+)$	$(O3^-, O3^-)$	$(\widetilde{O3}^-, O3^-)$
T	$(O3^+, \widetilde{O3}^-)$	$(O3^-, O3^+)$	$(\widetilde{O3}^-, \widetilde{O3}^-)$
T	$(O3^+, O3^-)$	$(O3^-, \widetilde{O3}^-)$	$(\widetilde{O3}^-, O3^+)$

where A stays for affine and T for twisted affine

Summary(I): electric models

Duality	**Gauge**	**Flavor**	**W**
Aharony	$U(N_c)_0$	(N_f, N_f)	W=0
GK	$U(N_c)_k$	(N_f, N_f)	W=0
ARSW	$U(N_c)_0$	(N_f, N_f)	$W = V_+ V_-$
(p,q)-BCC	$U(N_c)_{k>\|N_f - N_a\|/2}$	(N_f, N_a)	$W = 0$
(p,0)-BCC	$U(N_c)_{x/2}$	$(N_f, N_f - x)$	$W = 0$
(p,q)*-BCC	$U(N_c)_{k<\|N_f - N_a\|/2}$	(N_f, N_a)	$W = 0$
BBP	$U(N_c)_0$	(N_f, N_f)	$W = V_+ + V_-$
BBP	$U(N_c)_0$	(N_f, N_f)	$W = V_+$
BBP	$U(N_c)_{x/2}$	$(N_f, N_f - x)$	$W = V_+$
BBP	$U(N_c)_0$	(N_f, N_f)	$W = V_+^2 + V_-^2$
AGM	$U(N_c)_0$	(N_f, N_f)	$W = V_+^2$
AGM	$U(N_c)_{x/2}$	$(N_f, N_f - x)$	$W = V_+^2$
ACGM	$U(N_c)_0$	(N_f, N_f)	$W = V_+^2 + V_-$

Summary(II):magnetic models

Duality	Gauge	W
Aharony	$U(N_f - N_c)_0$	$W = Mq\tilde{q} + v_+ V_+ + v_- V_-$
GK	$U(N_f - N_c + \|k\|)_{-k}$	$W = Mq\tilde{q}$
ARSW	$U(N_c)_0$	$W = Mq\tilde{q} + v_+ v_-$
(p,q)-BCC	$U(\frac{N_f+N_a}{2} - N_c + \|k\|)_{-k}$	$W = Mq\tilde{q}$
(p,0)-BCC	$U(N_f - N_c)_{-x/2}$	$W = Mq\tilde{q} + v_+ V_+$
(p,q)*-BCC	$U(max(N_f, N_a) - N_c)_{-k}$	$W = Mq\tilde{q}$
BBP	$U(N_f - N_c - 2)_0$	$W = Mq\tilde{q} + v_+ + v_-$
BBP	$U(N_f - N_c - 1)_0$	$W = Mq\tilde{q} + v_+ + v_- V_-$
BBP	$U(N_f - N_c - 1)_{-x/2}$	$W = Mq\tilde{q} + v_+$
BBP	$U(N_f - N_c)_0$	$W = Mq\tilde{q} + v_+^2 + v_-^2$
AGM	$U(N_f - N_c)_0$	$W = Mq\tilde{q} + v_+^2 + v_- V_-$
AGM	$U(N_f - N_c)_{-x/2}$	$W = Mq\tilde{q} + v_+^2$
ACGM	$U(N_f - N_c - 1)_0$	$W = Mq\tilde{q} + v_+^2 + v_-$

Conclusions

- Rich web of dualities for $U(N_c)$ SQCD thanks to dimensional reduction, real mass flows, Higgs flows and monopole superpotentials;
- The web can in principle be enlarged (similar discussions hold in the non-supersymmetric case);
- Higher powers for the monopoles are forbidden (in absence of extra charged matter fields);
- Fractional powers are not forbidden. Do they have any field theory interpretation?
- Generalizations to $SU(N_c)$, $USp(2N_c)$ and $SO(N_c)$, quivers and tensorial matter.