Microstate counting for AdS black holes

Alberto Zaffaroni

Milano-Bicocca

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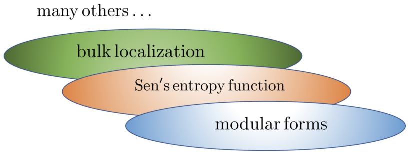
Introduction

A major achievement of string theory is the counting of micro-states for a class of asymptotically flat black holes [Vafa-Strominger'96]

- The entropy is obtained by counting states in the corresponding string/D-brane system

- Remarkable precision tests including higher derivatives



No similar results for asymptotically AdS_4 or AdS_5 black holes until very recently.

Introduction

Recent progress

- initiated with static magnetically charged black holes in $\mathrm{AdS}_4 \times S^7$

[Benini-Hristov-AZ, 2015]

- continued for electrically charged and rotating black holes in $\mathrm{AdS}_5 \times S^5$ with results in various overlapping limits

[Choi,Kim,Kim,Naamgoong, 2018]

[Cabo-Bizet,Cassani,Martelli,Murthy, 2018]

[Benini-Milan, 2018]

These results have been obtained through localisation and have been extended to other compactifications and dimensions.

Field Theory Perspective

In field theory we compute a grancanonical partition function

$$Z(\Delta_I,\omega_i) = \mathrm{Tr}\left((-1)^F e^{i(Q_I\Delta_I+J_i\omega_I)}e^{-\beta H}\right) = \sum_{\mathfrak{q},\mathfrak{j}} e^{S(\mathfrak{q},\mathfrak{j})}e^{i(\mathfrak{q}_I\Delta_I+\mathfrak{j}_i\omega_i)}$$

topologically twisted or superconformal index

The entropy $S(\mathfrak{q},\mathfrak{j})$ of a black hole with charge $\mathfrak{q}$ and angular momentum $\mathfrak{j}$ in a saddle point approximation is a Legendre Transform

$$\boxed{S_{BH}(\mathfrak{q},\mathfrak{j}) \equiv \mathcal{I}(\Delta,\omega) = \log Z(\Delta_I,\omega_i) - i(\mathfrak{q}_I\Delta_I + \mathfrak{j}_i\omega_i)} \qquad \frac{d\mathcal{I}}{d\Delta} = \frac{d\mathcal{I}}{d\omega} = 0$$

sometimes referred as $\mathcal{I}$-extremization for magnetically charged black holes

Dual Field Theory Perspective

The partition function is exactly computable only in the supersymmetric case

- $Z^{\text{susy}}_{S^{d-2}\times S^1}(\Delta_I, \omega_i) = \mathrm{Tr}\left((-1)^F e^{i(Q_I\Delta_I + J_i\omega_I)} e^{-\beta H_{\mathfrak{p}}}\right)$
- cancellation between massive boson and fermions (Witten index)
- sum over supersymmetric ground states $H_{\mathfrak{p}} = 0$;

What's about $(-1)^F$? we assume no cancellation between bosonic and fermionic ground states. Seems to be true in the limit of large charges.

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t\gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

$$\partial_t Z = \int \{Q, V\} e^{-S+t\{Q,V\}} = 0$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

Localization

Localization ideas apply to path integral of Euclidean supersymmetric theories

- Compact space provides IR cut-off, making path integral well defined
- Localization reduces it to a finite dimensional integral, a matrix model

$$\int \prod_{i=1}^{N_1} du_i \prod_{j=1}^{N_2} dv_j \frac{\prod_{i<j} \sinh^2 \frac{u_i-u_j}{2} \sinh^2 \frac{v_i-v_j}{2}}{\prod_{i<j} \cosh^2 \frac{u_i-v_j}{2}} e^{\frac{ik}{4\pi}\left(\sum u_i^2 - \sum v_j^2\right)}$$

ABJM, 3d Chern-Simon theories, [Kapustin,Willet,Yakoov;Drukker,Marino,Putrov]

Localization

Carried out recently in many cases

- many papers on topological theories
- S^2, T^2
- S^3, $S^3/\mathbb{Z}_k$, $S^2 \times S^1$, Seifert manifolds
- S^4, $S^4/\mathbb{Z}_k$, $S^3 \times S^1$, ellipsoids
- S^5, $S^4 \times S^1$, Sasaki-Einstein manifolds

with addition of boundaries, codimension-2 operators, . . .

Pestun 07; Kapustin,Willet,Yakoov; Kim; Jafferis; Hama,Hosomichi,Lee, too many to count them all · · ·

Localization

In all cases, it reduces to a finite-dimensional matrix model on gauge variables, possibly summed over different topological sectors

$$Z_M(y) = \sum_{\mathfrak{m}} \int_C dx \, Z_{\text{int}}(x, y; \mathfrak{m})$$

with different integrands and integration contours.

When backgrounds for flavor symmetries are introduced, $Z_M(y)$ becomes an interesting and complicated function of y which can be used to test dualities

- Sphere partition function, Kapustin-Willet-Yakoov; $\cdots$
- Superconformal index, Spironov-Vartanov; Gadde,Rastelli,Razamat,Yan; $\cdots$
- Topologically twisted index, Benini,AZ; Closset-Kim; $\cdots$

Entropy functional

In gravity we typically define an entropy functional [Ferrara-Kallosh-Strominger 97; OSV 04; Sen 05]

$$\boxed{\mathcal{I}(X_I,\Omega_i) = \mathcal{E}(X_I,\Omega_i) - i(\mathfrak{q}_I X_I + \mathfrak{j}_i \Omega_i)}$$

depending on the gravity scalar fields and other modes whose extremization realises the attractor mechanism :

$$S_{BH}(\mathfrak{q},\mathfrak{j}) \equiv \mathcal{I}(\bar{X}_I,\bar{\Omega}_i)\Big|_{\text{crit}}$$
$$\bar{X}_I,\bar{\Omega}_i \equiv \text{horizon value}$$

Comparison

The two pictures are expected to match:

- dyonic static $AdS_4 \times S^7$ black holes: QFT computation = attractor mechanism in $\mathcal{N} = 2$ gauged supergravity [Ferrara-Kallosh-Strominger 96; Dall'Agata-Gnecchi 10]

Not always the attractor mechanism is known: entropy functional can be written combining field theory and gravity intuition

- Kerr-Newman $AdS_5 \times S^5$: entropy functional found empirically [Hristov-Hosseini-AZ, 17]

$$\mathcal{I}(\Delta_a, \omega_i) = i\pi N^2 \frac{\Delta^1\Delta^2\Delta^3}{\omega_1\omega_2} + 2\pi i \left(\sum_{a=1}^{3} \Delta^a Q_a - \sum_{i=1}^{2} \omega_i J_i \right), \qquad \Delta^1 + \Delta^2 + \Delta^3 + \omega_1 + \omega_2 = 1$$

– Reproduced in QFT in various overlapping limits using the superconformal index [Choi,Hwang,Kim,Nahmgoong;Cabo-Bizet,Cassani,Martelli,Murthy; Benini-Milan;Cabo-Bizet-Murthy]

– Similar functionals proposed in higher dimensions and also computed via on-shell actions [Hristov-Hosseini-AZ;Choi,Hwang,Kim,Nahmgoong;Cabo-Bizet,Cassani,Martelli,Murthy; Cassani-Papini]

Example I: Static black holes in $AdS_4 \times S^7$

Black holes in M theory on $AdS_4 \times S^7$: [Cacciatori, Klemm 08; Dall'Agata, Gnecchi; Hristov, Vandoren 10; Katmadas; Halmagyi 14; Hristov, Katmadas, Toldo 18]

- preserves two real supercharges (1/16 BPS) and horizon $AdS_2 \times \Sigma_g$
- four electric $\mathfrak{q}_a$ and magnetic $\mathfrak{p}_a$ charges under $U(1)^4 \subset SO(8)$; only six independent parameters
- supersymmetry preserved with a topological twist
- entropy goes like $O(N^{3/2})$ and is a complicated function

$$S_{\rm BH}(\mathfrak{p}_a, \mathfrak{q}_a) \sim \sqrt{I_4(\Gamma,\Gamma,G,G) \pm \sqrt{I_4(\Gamma,\Gamma,G,G)^2 - 64 I_4(\Gamma) I_4(G)}}$$

I_4 symplectic quartic invariant

$\Gamma = (\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4)$ [Halmagyi 13]

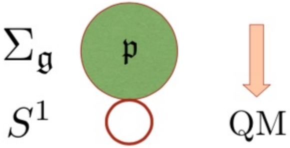
$G = (0, 0, 0, 0, g, g, g, g)$

The relevant index

Topologically twisted index = QM Witten index

$$Z_{\Sigma_\mathfrak{g}\times S^1}(\Delta_I,\mathfrak{p}_a) = \underbrace{\mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{i\sum_{a=1}^4 Q_a\Delta_a} e^{-\beta H_\mathfrak{p}}\right)}_{\sum_{a=1}^4 \Delta_a \in 2\pi\mathbb{Z}}$$

- magnetic charges $\mathfrak{p}_a$ enter in the Hamiltonian $H_\mathfrak{g}$, electric charges $\mathfrak{q}_a$ introduced through chemical potentials Δ_a
- number of fugacities equal to the number of conserved charges



ABJM twisted index

Luckily enough, the topologically twisted partition function for ABJM can be evaluated using localization

$$Z^{\text{susy}}_{S^2\times S^1} = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}}\in\mathbb{Z}^N} \int \prod_{i=1}^N \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k\mathfrak{m}_i} \tilde{x}_i^{-k\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^N \left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)\times$$

$$\times \prod_{i,j=1}^N \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_1}}{1-\frac{x_i}{\tilde{x}_j} y_1}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{p}_1+1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_2}}{1-\frac{x_i}{\tilde{x}_j} y_2}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{p}_2+1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_3}}{1-\frac{\tilde{x}_j}{x_i} y_3}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_3+1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_4}}{1-\frac{\tilde{x}_j}{x_i} y_4}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{p}_4+1}$$

$$\prod_a y_a = 1\,, \qquad \sum \mathfrak{p}_a = 2$$

and solved in the large N limit. There is no cancellation between bosons and fermions and $\log Z = O(N^{3/2})$. [Benini-AZ; Benini-Hristov-AZ]

QFT/Gravity comparison

- dyonic static AdS$_4 \times S^7$ black holes: QFT computation = attractor mechanism in $\mathcal{N} = 2$ gauged supergravity

Entropy functional: [Ferrara-Kallosh-Strominger 96; Dall'Agata-Gnecchi 10]

$$S_{\rm BH}(\mathfrak{p}_a, \mathfrak{q}_a) = \log Z(X_a, \mathfrak{p}_a) - \sum_a iX_a \mathfrak{q}_a \Big|_{crit} = \sum\nolimits_a i\mathfrak{p}_a \frac{\partial \mathcal{F}}{\partial X_a} - iX_a \mathfrak{q}_a \Big|_{crit}$$

gauged supergravity prepotential $\mathcal{F} \sim \sqrt{X_1 X_2 X_3 X_4}$

$\sum X_a = 2\pi$ horizon scalar fields

Localization (topologically twisted index): [Benini,Hristov,AZ 05]

$$S(\mathfrak{p}_a, \mathfrak{q}_a) = \log Z(\Delta_a, \mathfrak{p}_a) - \sum_a i\Delta_a \mathfrak{q}_a \Big|_{crit} = \sum\nolimits_a i\mathfrak{p}_a \frac{\partial \mathcal{W}}{\partial \Delta_a} - i\Delta_a \mathfrak{q}_a \Big|_{crit}$$

twisted superpotential $\mathcal{W}_{\rm on-shell} = \frac{2}{3} i N^{3/2} \sqrt{2\Delta_1 \Delta_2 \Delta_3 \Delta_4}$

$\sum_{a=1}^4 \Delta_a = 2\pi \qquad {\rm Re}\Delta_a \in [0, 2\pi]$

Example II: Rotating black holes in $AdS_5 \times S^5$

Most famous BPS examples are asymptotic to $AdS_5 \times S^5$

two angular momenta J_1, J_2 in AdS_5 $\quad U(1)^2 \subset SO(4) \subset SO(2,4)$

three electric charges Q_I in S^5 $\quad U(1)^3 \subset SO(6)$

with a constraint $F(J_i, Q_I) = 0$. They must rotate and preserves two supercharges.

$$S_{\rm BH} = 2\pi\sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - 2c(J_1 + J_2)} \qquad c = \frac{N^2-1}{4}$$

[Gutowski-Reall 04; Chong, Cvetic, Lu, Pope 05; Kunduri, Lucietti, Reall; Kim, Lee, 06]

The boundary metric is $S^3 \times \mathbb{R}$, no twist. The microstates correspond to states of given angular momentum and electric charge in $\mathcal{N} = 4$ SYM.

Recent examples of hairy black holes with more parameters [Markeviciute, Santos 18]

Entropy function for AdS$_5$ black holes

- BPS entropy function [Hosseini,Hristov,AZ 17]

$$\mathcal{S}_{\rm BH}(Q_I, J_i) = -i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} - 2\pi i\left(\sum_{I=1}^{3} Q_I\Delta_I + \sum_{i=1}^{2} J_i\omega_i\right)\Bigg|_{\bar\Delta_I,\bar\omega_i}$$

with $\Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2=\pm 1$

- From BH thermodynamics: chemical potentials $\bar\Delta_I, \bar\omega_i$ can be obtained in a suitable zero-temperature limit for a family of supersymmetric Euclidean black holes [Cabo-Bizet, Cassani, Martelli, Murthy 18]

$$-i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} = \text{on-shell action}$$

The critical values $\bar\Delta_I, \bar\omega_i$ are complex but, quite remarkably, the extremum is a *real* function of the black hole charges.

Field Theory Comparison

Entropy scales like $O(N^2)$ for $Q_I, J_i \sim N^2$. The superconformal index

$$\mathrm{Tr}(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{2\pi i(\Delta_I Q_I + \omega_i J_i)} = \oint \frac{dz_i}{2\pi i z_i} \prod_{1\leq i<j\leq N} \frac{\prod_{k=1}^3 \Gamma_e(y_k (z_i/z_j)^{\pm 1}; p, q)}{\Gamma_e((z_i/z_j)^{\pm 1}; p, q)}$$

- For real fugacities: $\log Z = O(1)$. Large cancellations between bosons and fermions. Long standing puzzle [Kinney, Maldacena, Minwalla, Raju 05]
- For complex fugacities (like the ones in sugra) is consistent with

$$\log Z(\Delta, \omega) \sim -i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = \pm 1$$

[Cardy limit $\omega_i \ll 1$: Choi, Kim, Kim, Nahmgoong]

[Modified index/partition function: Cabo-Bizet, Cassani, Martelli, Murthy]

[Large N and $J_1 = J_2$: Benini, Milan 18; Cabo-Bizet, Murthy 19]

General entropy functional

$\mathcal{N} = 2$ gauged supergravity coupled to vector multiplet is specified by

symplectic section	X^Λ
prepotential	$\mathcal{F}(X)$
gaugings	$\{g^\Lambda, g_\Lambda\}$

and covariant under symplectic transformations (electric/magnetic dualities)

All black objects in 4d or 5d can be dimensionally reduced to a 4d black hole with

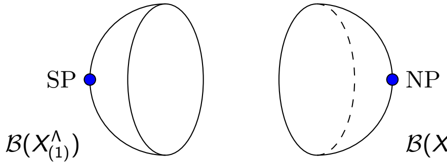
angular momentum	$\mathcal{J}$
magnetic and electric charges	$\{p^\Lambda, q_\Lambda\}$

Gravitational blocks

The entropy functional can be obtained by gluing gravitational blocks

$$\mathcal{I}(p^\Lambda, \chi^\Lambda, \omega) \equiv \frac{\pi}{4G_{\rm N}^{(4)}} \left(\sum_{\sigma=1}^{2} \mathcal{B}(X^\Lambda_{(\sigma)}, \omega_{(\sigma)}) - 2i\chi^\Lambda q_\Lambda - 2\omega\mathcal{J} \right)$$

$$\boxed{\mathcal{B}(X^\Lambda, \omega) = -\frac{\mathcal{F}(X^\Lambda)}{\omega}}$$



with

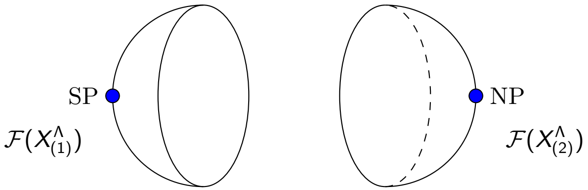
A-gluing

$$X^\Lambda_{(1)} = \chi^\Lambda - i\omega p^\Lambda\,, \qquad \omega_{(1)} = \omega$$
$$X^\Lambda_{(2)} = \chi^\Lambda + i\omega p^\Lambda\,, \qquad \omega_{(2)} = -\omega$$

id-gluing

$$X^\Lambda_{(1)} = \chi^\Lambda - i\omega p^\Lambda\,, \qquad \omega_{(1)} = \omega$$
$$X^\Lambda_{(2)} = \chi^\Lambda + i\omega p^\Lambda\,, \qquad \omega_{(2)} = \omega$$

Attractor Mechanism



Extremize $\mathcal{I}$:

$$S_{\text{BH}}^{\text{AdS}_5\,\text{BS}}(\mathfrak{p}^i, q_\Lambda, \mathcal{J}) = \mathcal{I}_{\text{AdS}_5\,\text{BS}}(\mathfrak{p}^i, \chi^\Lambda, \omega)\Big|_{\text{crit.}}$$

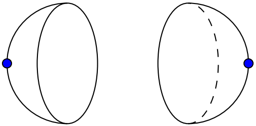
Attractor mechanism:

$$X^\Lambda_{\text{SP,NP}} \qquad \Longrightarrow \qquad X^\Lambda_{(\sigma)} = \chi^\Lambda \mp i\omega p^\Lambda\Big|_{\text{crit.}}$$

Holomorphic blocks

Our construction is inspired by holomorphic blocks in SQFT and factorization of partition functions [Beem,Dimofte,Pasquetti 12]

$\mathcal{B}^\alpha(X^\Lambda_{(1)}, \omega_{(1)})$



$\mathcal{B}^\alpha(X^\Lambda_{(2)}, \omega_{(2)})$

$$Z(\Delta^\Lambda|\omega) = \sum_\alpha B^\alpha(\Delta^\Lambda_{(1)}|\omega_{(1)}) B^\alpha(\Delta^\Lambda_{(2)}|\omega_{(2)})$$

In the Cardy limit

$$B^\alpha(\Delta^\Lambda|\omega) \underset{\omega\to 0}{\sim} \exp\left(-\frac{1}{\omega}\mathcal{W}(x^\alpha, \Delta^\Lambda)\right)$$

it has been observed that, on the relevant vacuum at large N [Hosseini,(Nedelin),AZ;06]

$$\mathcal{W}(x^\alpha, \Delta^\Lambda)\Big|_{\text{BA}} \equiv \widetilde{\mathcal{W}}(\Delta^\Lambda) = \mathcal{F}(X^\Lambda)$$

What we can learn about near BPS black holes?

I. We can try to derive entropy functionals in gravity [see for example Larsen,Nian,Zheng 19]

- Extremal non-supersymmetric black holes, $T = 0$

  Attractor mechanism, fake superpotentials... [Kallosh; Ferrara et al; Dall'Agata-Ceresole; Gnecchi-Toldo]

- Near extremal black holes, $T \ll 1$

$$S = S_{\rm extr} + \gamma T$$

γ universal and related to Schwarzian description, SYK etc...

What we can learn about near BPS black holes?

II. We can try to compare the two QFT pictures we know

- a supersymmetric quantum mechanics with many vacua describing the horizon of BPS black hole
- SYK or melonic tensor models describing near AdS_2 physics

Deforming localization can be difficult. Finding an effective description of the supersymmetric QM where to turn on T could be simpler.

Gravitational Blocks

$$\mathcal{I}(p^\Lambda, \chi^\Lambda, \omega) \equiv \frac{\pi}{4G_N^{(4)}} \left(\sum_{\sigma=1}^{2} \mathcal{B}(X^\Lambda_{(\sigma)}, \omega_{(\sigma)}) - 2i\chi^\Lambda q_\Lambda - 2\omega\mathcal{J} \right)$$

$$\mathcal{B}(X^\Lambda, \omega) \equiv -\frac{\mathcal{F}(X^\Lambda)}{\omega}$$

Black object	Gluing	Constraint	$\mathcal{F}(X^\Lambda)$	g_Λ
$m\mathrm{AdS}_4$	A-gluing	$g_\Lambda\chi^\Lambda = 2$	$2i\sqrt{X^0X^1X^2X^3}$	$\{1,1,1,1\}$
AdS_5 BS	A-gluing	$g_\Lambda\chi^\Lambda = 2$	$\frac{X^1X^2X^3}{X^0}$	$\{1,1,1,1\}$
KN-AdS_4	id-gluing	$g_\Lambda\chi^\Lambda - i\omega = 2$	$2i\sqrt{X^0X^1X^2X^3}$	$\{0,1,1,1\}$
KN-AdS_5	A-gluing	$g_\Lambda\chi^\Lambda - i\omega\tanh(\delta) = 2$	$\frac{X^1X^2X^3}{X^0}$	$\sqrt{2}\{\cosh(\delta),1,1,1\}$